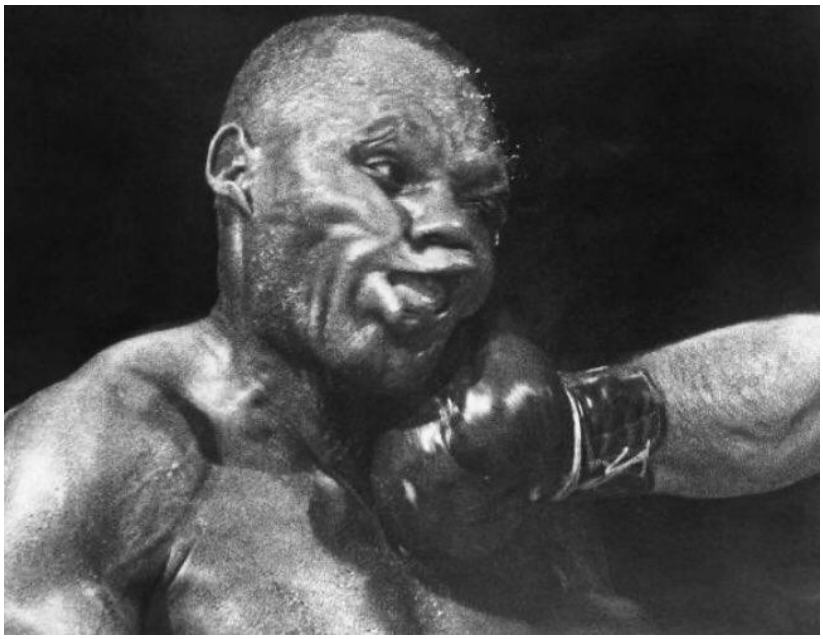

NOVEL QUESTIONS FOR OLD ANSWERS

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My teachers used to tell me: "Tozzi, be on focus". But I never liked "to be on focus": I like a lot to play with hidden, unexpected multidisciplinary relationships. Despite my harsh criticism to current attitudes toward scientific issues, I never take a full sceptic turn: I engage in "pragmaticistic" active outreach, showing how fresh interpretation of everlasting questions sheds new light on countless scientific issues. Here I try to provide scientific… questions to classical answers through the unusual format of the Quodlibets. The Medieval quodlibetal (quodlibet= any whatever) questions consisted in raising issues or objections about anything from basic Christian issues to heretic controversies. Open to a broader public - masters, students from different schools, ecclesiastical and civil authorities - these questions could be posed by any member of the audience without any prior notice. While Medieval philosophers favoured a methodology consisting of the Aristotelian deductive logic, I will use a twofold approach to my quodlibetal questions: 1) the Galilean inductive method distinctive of the current scientific attitude; 2) a metatheoretic starting point that I term "testable rationalism": sharp experimental previsions arising from top-down, deductive mathematical/topological approaches. In this paper, going through physical, biological, neuroscientific and philosophical issues, I do hope that the reader will forgive me.

N.B: a long list of keywords can be found in the footnotes.

…and, of course, thanks to my friend James Peters and his enthusiasm.

IS IT FEASIBLE VIA MAPPINGS AND PROJECTIONS TO CORRELATE TOPOLOGICAL ABSTRACTIONS WITH THE BIOPHYSICAL WORLD?

Projectionism & brain manifolds: the philosophy beyond our approach[1]. We introduce a novel methodological approach, termed "projectionism", to assess mappings and projections among different functional biophysical dimensions and phase spaces. Our main focus concerns neuroscience. We describe recently published papers that confirm our general framework and compare brain symmetries with the predictive coding standing for a sort of Kant *a priori* located in our brains. We illustrate the "unreasonable power" of topology in neuroscience: this allows a rationalistic but testable top-down inquiry of the brain activity, to mathematically assess the physical and biological dynamics of the human nervous system. We also propose possible biochemical correlates of a brain fourth dimension, with clues provided by… LSD intake. Also, we suggest fresh mathematical approaches to brain topological dynamics, introducing novel theorems and proposing complex functional nervous spaces very different from the classical Euclidean ones. Also, we provide a novel computational scenario that takes into account the tenets of neural Darwinism.

Hidden symmetries (and symmetry breaks): entering the Borsuk-Ulam theorem as a general principle in biophysical issues[2]. Symmetries are widespread invariances underlining countless systems. A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level. In such a general framework, a concept from algebraic topology, namely the Borsuk-Ulam theorem (BUT), comes into play and sheds new light on the general mechanisms of nervous symmetries. BUT tells us that we can find, on an n-dimensional sphere, a pair of opposite points that have same encoding on an n-1 sphere. This mapping makes it possible to describe both antipodal points with a single real-valued vector on a lower dimensional sphere. Here we argue that this topological approach is useful in the evaluation of hidden biophysical symmetries. This means that symmetries can be found when evaluating the system in a proper dimension, while they disappear (are hidden or broken) when we evaluate the same system in just one dimension lower.

A topological causality?[3] Observable facts involving changes in system shape, dimension and symmetry may elude simple cause and effect inductive explanations. We argue that numerous physical and biological phenomena such as chaotic dynamics, symmetry breaking, long-range collisionless neural interactions, zero-valued energy singularities, and particle/wave duality can be accounted for in terms of purely topological mechanisms devoid of causality. We illustrate how simple topological theorems, seemingly far away from scientific inquiry (e.g., "given at least some wind on Earth, there must at all times be a cyclone or anticyclone somewhere"; "if one stirs to dissolve a lump of sugar in a cup of coffee, it appears there is always a point without motion"; "at any moment, there is always a pair of antipodal points on the Earth's surface with equal temperatures and barometric pressures") reflect the action of non-causal topological rules. To do so, we introduce some fundamental topological tools and illustrate how phenomena such as double slit

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[1] *Tozzi A, Peters JF, Fingelkurts AA, Fingelkurts AA, Marijuán PC. 2017. Brain projective reality: novel clothes for the emperor. Reply to comments on "Topodynamics of metastable brains" by Tozzi et al. Physics of Life Reviews, 21, 46-55. https://doi.org/10.1016/j.plrev.2017.06.020.*

Cech complexes, Costa minimal surface, Oloid surface, De Raham cohomology; physics, complex nervous phase spaces; neuroscience, multidimensional brain, Neuroscience themes (from British Neuroscience Society), BUT in neurons, Multidimensional brain, LSD, Hominids' large brains, neural darwinism; Projectionism, Projective reality (rather than causality), Projections vs causation, Kant's a priori, Mathematical realism, Wigner, Darwin; Pragmatism, metatheory; top-down deductive approaches, testable rationalism, experimental prevision

[2] *Tozzi A, Peters JF. 2016. A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain. Journal of Neuroscience Research 94 (5): 351–65. doi:10.1002/jnr.23720..*

Grassmann manifolds, Homotopies, Delaunay tessellations, Re-BUT, exponential-BUT, time-BUT, concave-BUT, nonlinear-BUT, fractal-BUT, oscillating-BUT; physics, Bougin-Yang-type theorems, Hidden symmetries, Broken symmetries, Hausdorff measure, Power laws, Variational free-energy, Hullback-Leibler divergence; neuroscience, multidimensional brain, Quantum mind theories, Chaotic brain, Tessellations in brain images, Temporal dimensions, Multisensory integration, Scale-invariant behavior, Oscillating manifolds; Circular continuum

[3] *Tozzi A, Papo D. 2020. Projective mechanisms subtending real world phenomena wipe away cause effect relationships. Progress in Biophysics and Molecular Biology. 151:1-13. DOI: 10.1016/j.pbiomolbio.2019.12.002.*

Tozzi A. 2019. RE: towards applications for exceptional points also in non-hermitian biological networks. (electronic response to: Miri M-A, Alù A. 2019. Exceptional points in optics and photonics. Science, 363 (6422), eaar7709. DOI: 10.1126/science.aar7709.

Kneser graphs, Patch connected subsets, Ham sandwich theorem, Brower fixed point theorem, Wired friend theorem, Lusternik-Schnirelmann theorem, Hairy ball theorem, Ulenbeck equations, Eigenvalue surfaces, Exceptional points, operational-BUT, BUT as universal principle, Split-BUT; fragile topology, magic angle, superconductor, graphene; physics, Causality, Entropy, Shannon plot, Double slit experiment, Particle/wave duality, Gravitational lensing, Plasma collisionless movements, Ergodicity, II law thermodynamics, Non-Hermitian systems, Cavity-laser sources, information; Adverse drug reactions; Mytosis, Ecological approach to visual perception, Cellular membrane, Dewetting transition, Social processes, Intestinal organoids, Viral immunity, Singularities in CNS, E/I ratio, Imaginary values, Life definition, Inanimate matter's evolution, life origin; Causality, Projectionism, Feddoso, Al-Ghazali, Phyrrho, Mirecourt, Autrecourt, Berkeley, Hume, Montaigne, Feyerabend, Fogelikn, Van Fraassen, Gibson, Rejection of continuous, Spencer, Tyler, Teleology of life, passivity

experiments, cellular mechanisms and some aspects of brain function can be explained in terms of geometric projections and mappings, rather than local physical effects. We conclude that unavoidable, passive, spontaneous topological modifications may lead to novel functional biophysical features, independent of exerted physical forces, thermodynamic constraints, temporal correlations and probabilistic a priori knowledge of previous cases.

A novel methodology (with examples): investigating objects from outside[4]. Set theory faces two difficulties: formal definitions of sets/subsets are incapable of assessing biophysical issues; formal axiomatic systems are complete/inconsistent or incomplete/consistent, due to the Godel's theorems. To overtake these problems reminiscent of the old-fashioned principle of individuation, we provide formal treatment/validation/operationalization of a methodological weapon termed "outer approach" (OA). The observer's attention shifts from the system under evaluation to its surroundings, so that objects are investigated from outside. Subsets become just "holes" devoid of information inside larger sets. Sets are no longer passive containers, rather active structures enabling their content's examination. Consequences/applications of OA include: a) operationalization of paraconsistent logics, anticipated by unexpected forerunners such as Nicholas od Autrecourt and Richard Avenarius, in terms of advanced truth theories of natural language, anthropic principle and quantum dynamics; b) assessment of Hirschsprung's disease and embryonic craniocaudal migration in terms of Turing's spots; c) evaluation of hominids' social behaviors in terms of evolutionary modifications of facial expression's musculature; d) treatment of cortical action potentials in terms of collective movements of extracellular currents, leaving apart what happens inside the neurons; e) a critique of Shannon's information in terms of the Arabic thinkers' active/potential intellect; f) novel interpretation of medical issues such as the use of oral propranolol in infantile hemangioma. Also, OA provides an outer view of a) humanistic issues such as the enigmatic Celestino of Verona's letter, Dante Alighieri's Vth Chant of "Hell" and the puzzling Voynich manuscript; b) historical issues such as Aldo Moro's death and the Liston/Clay boxing fight. Summarizing, the safest methodology to quantify phenomena is to remove them from our observation and tackle an outer view, since mathematical/logical issues such as selective information deletion and set complement rescue incompleteness/inconsistency of biophysical systems.

Tozzi A. 2015. Oral Propranolol for Infantile Hemangioma. The New England Journal of Medicine 373 (3): 284–85. doi:10.1056/NEJMc1505699.

IS THERE STILL ROOM FOR NEW VERSIONS OF THE BORSUK-ULAM THEOREM AND OTHER TOPOLOGICAL ISSUES?

Borsuk-Ulam Theorem (BUT) variants[5]:
a) BUT on concave manifolds.
b) Region-based extension of BUT. We introduce a region-based extension of the Borsuk-Ulam Theorem (denoted by reBUT). A region is a subset of a surface on a finite-dimensional n-sphere. In topology, an n-sphere is a generalization of the circle. For a continuous function on an n-sphere into n-dimensional Euclidean space, there exists a pair of antipodal n-sphere regions with matching

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[4] *Tozzi A. 2021. To Know Them, Remove Them: An Outer Methodological Approach to Biophysics and Humanities. Preprints. doi: 10.20944/preprints202104.0681.v1.*
Tozzi A. 2018. Dante Alighieri's "Amor, ch'a nullo Amato…": an "ex nihilo" account of "a nullo". viXra:1811.0287.
Tozzi A. 2018. Sonny Liston and the torn tendon. SportRxiv. September 6. doi:10.31236/osf.io/2p7w6.
Peters JF; Tozzi A. 2018. Computational Topology Techniques Help to Solve a Long-Lasting Forensic Dilemma: Aldo Moro's Death. Preprints 2018, 2018110310 (doi: 10.20944/preprints201811.0310.v1).
Tozzi A. 2021. Nicholas of Autrecourt: A Forerunner of Paraconsistent Logics. Preprints 2021, 2021030238. doi: 10.20944/preprints202103.0238.v1.
God as first cause, Parmenides, Plotinus, Amor Cortese, Cappellano, 1277 condemnations,
Sports medicine, Orthopedics, Cassius Clay
Vortex cycles, Nerves, Alexandroff nerves, connectedness proximity; Forensic medicine, Autopsy; Aldo Moro

Tozzi A. 2018. Reaction-diffusion mechanisms underlying Hirschprung's disease and their practical impications. BioRxiv, doi: https://doi.org/10.1101/492884.
Turing's reaction/diffusion; Hirschsprung, Pull-through operation, gut development, neural crest, neuroblasts, HD mutations, Pediatrics; Turing

Tozzi A. 2019. Brain Tissue Viscolasticity & EEG Oscillations: Towards Novel Drugs for Alzheimer's Disease. viXra:1901.0050.
Deborah number, Rheometry, Viscoelasticity, Stiffness; neuroscience, brain, Alzheimer, Dementia, Tromboelastogram, mannitol, Pharmacology, EEG waves

Tozzi A. 2016. Muscles of facial expression in extinct species of the genus Homo. bioRxiv 072884; doi: http://dx.doi.org/10.1101/072884.
Mimetic muscles, Hominids casts, sapiens, neanderthalensis, erectus, heidelbergensis, ergaster

[5] *Tozzi A. 2016. Borsuk-Ulam Theorem Extended to Hyperbolic Spaces. In Computational Proximity. Excursions in the Topology of Digital Images, edited by J F Peters, 169–171. doi:10.1007/978-3-319-30262-1.*
Peters JF, Tozzi A. 2016. Region-Based Borsuk-Ulam Theorem. arXiv.1605.02987.
Peters JF, Tozzi A. 2016. String-Based Borsuk-Ulam Theorem. arXiv:1606.04031.
Lodato proximity, Strongly near sets, concave-BUT; hyperbolic, negative curvature, Re-BUT, Straecker digital BUT, Burak-Karaca digital BUT, bounded shapes projection mapping, wired friend theorem, torus, string-BUT, worldsheet; Worldline,Worldsheet; EEG scenarios, BUT in brain

descriptions that map into Euclidean space Rn. The main results include a number of different region-based forms of the classical Borsuk-Ulam Theorem as well as the Straecker digital Borsuk-Ulam Theorem and the Burak-Karaca digital Borsuk-Ulam Theorem. Applications of reBUT are given in the evaluation of brain activity and quantum entanglement.

c) String-based extension of BUT. We introduce a string-based extension of the Borsuk-Ulam Theorem (denoted by strBUT). A string is a region with zero width and either bounded or unbounded length on the surface of an n-sphere or a region of a normed linear space. In this work, an n-sphere surface is covered by a collection of strings. For a strongly proximal continuous function on an n-sphere into n-dimensional Euclidean space, there exists a pair of antipodal n-sphere strings with matching descriptions that map into Euclidean space Rn. Each region M of a string-covered n-sphere is a worldsheet (denoted by wshM). For a strongly proximal continuous mapping from a worldsheet covered n-sphere to Rn, strongly near antipodal worldsheets map into the same region in Rn. An application of strBUT is given in terms of the evaluation of Electroencephalography (EEG) patterns.

Deformation is not a topological invariant: a critique to topology[6]. Topology deals with the properties of space preserved under continuous deformations, such as stretching, twisting, bending and so on. This means that two shapes of genus zero (or one, or two, and so on) are topologically invariant under homeomorphisms, i.e., they share matching topological description. Here we ask: is this tenet true? Take a positive-curvature active surface, such as a spherical soap bubble. Due to the Borsuk-Ulam theorem, the bubble's surface displays at least two antipodal points with the same description (e.g., two antipodal points with the same value of surface tension, the latter standing for a continuous function on the 2D surface of the 3D bubble). When a spontaneous or a mechanical stress (e.g., an internal or external force, or a torque) is applied within and onto the surface, the subsequent instability leads to the production of a deformed bubble. The formation of this bubble's nontrivial surface shape leads to the loss of the above-mentioned antipodal points with matching description. Therefore, once a spherical manifold's curvature is modified, an algebraic topological feature gets lost, i.e., the two antipodal points with matching description.

A fully novel topological theorem[7]. Take a balloon, that is a genus-one manifold. If you break the jointness by piercing its surface, the hole gest lost and the punctured balloon becomes a genus-0 manifold. Starting from this trivial claim, we prove a topological theorem which plainly states that "the ends of a donut can meet, whilst the ends of a kidney pie cannot". We discuss the theorem and its implications in disparate topics such as topological connectedness, gauge theories and the physics of the black holes.

DOES THE BRAIN ACTIVITY TAKE PLACE IN MULTIDIMENSIONAL SPACES?

Hidden symmetries (and symmetry breaks) in the brain: entering the Borsuk-Ulam theorem in the study of nervous issues[8]. Symmetries are widespread invariances underlining countless systems, including the brain. A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level. In such a general framework, a concept from algebraic topology, namely the Borsuk-Ulam theorem (BUT), comes into play and sheds new light on the general mechanisms of nervous symmetries. BUT tells us that we can find, on an n-dimensional sphere, a pair of opposite points that have same encoding on an n-1 sphere. This mapping makes it possible to describe both antipodal points with a single real-valued vector on a lower dimensional sphere. Here we argue that this topological approach is useful in the evaluation of hidden nervous symmetries. This means that symmetries can be found when evaluating the brain in a proper dimension, while they disappear (are hidden or broken) when we evaluate the same brain in just one dimension lower. We provide a topological methodology for the evaluation of the most general features of brain activity, i.e., the symmetries, cast in a physical/biological fashion that has the potential to be operationalized.

The brain activity takes place in higher dimensions: not just a figure of speech[9]! Brain activity takes place in three spatial-plus time dimensions. This rather

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[6] *Tozzi A. 2019. Is shape deformation a topological invariant? (electronic response to: Kawabata K, Higashikawa S, Gong Z, Ashida Y, Ueda M. 2019. Topological unification of time-reversal and particle-hole symmetries in non-Hermitian physics. Nature Communications 10: 297).*

Invariance under homeomorphisms, Abnormal-BUT on not-spherical Earth; properties preserved under continuous transformations; bubbles; surface tension

[7] *Tozzi A. 2021. Lack of Disjointness in Genus-1 Surfaces: The Punctured Balloon Theorem. viXra:2101.0005.*

Holes; black holes

[8] *Tozzi A, Peters JF. 2016. A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain. Journal of Neuroscience Research 94 (5): 351–65. doi:10.1002/jnr.23720..*

Grassmann manifolds, Homotopies, Delaunay tessellations, Re-BUT, exponential-BUT, time-BUT, concave-BUT, nonlinear-BUT, fractal-BUT, oscillating-BUT; physics, Bougin-Yang-type theorems, Hidden symmetries, Broken symmetries, Hausdorff measure, Power laws, Variational free-energy, Hullback-Leibler divergence; neuroscience, multidimensional brain, Quantum mind theories, Chaotic brain, Tessellations in brain images, Temporal dimensions, Multisensory integration, Scale-invariant behavior, Oscillating manifolds; Circular continuum

[9] *Tozzi A. 2019. The multidimensional brain. Physics of Life Reviews. doi: https://doi.org/10.1016/j.plrev.2018.12.004.*

obvious claim has been recently questioned by papers that, taking into account the big data outburst and novel available computational tools, are starting to unveil a more intricate state of affairs. Indeed, various brain activities and their correlated mental functions can be assessed in terms of trajectories embedded in phase spaces of dimensions higher than the canonical ones. Further dimensions may not just represent a convenient methodological tool that allows a better mathematical treatment of otherwise elusive cortical activities, rather may also reflect genuine functional or anatomical relationships among real nervous functions. I describe how to extract hidden multidimensional information from real or artificial neurodata series, and make clear how our mind dilutes, rather than concentrates as currently believed, inputs coming from the environment. Finally, I argue that the principle "the higher the dimension, the greater the information" may explain the occurrence of mental activities and elucidate the mechanisms of human diseases associated with dimensionality reduction.

The first clue towards the occurrence of a four-dimensional hypersphere in the brain[10]. Current advances in neurosciences deal with the functional architecture of the central nervous system, paving the way for general theories that improve our understanding of brain activity. From topology, a strong concept comes into play in understanding brain functions, namely, the 4D space of a "hypersphere's torus", undetectable by observers living in a 3D world. The torus may be compared with a video game with biplanes in aerial combat: when a biplane flies off one edge of gaming display, it does not crash but rather it comes back from the opposite edge of the screen. Our thoughts exhibit similar behaviour, i.e. the unique ability to connect past, present and future events in a single, coherent picture as if we were allowed to watch the three screens of past-present-future "glued" together in a mental kaleidoscope. Here we hypothesize that brain functions are embedded in a imperceptible fourth spatial dimension and propose a method to empirically assess its presence. Neuroimaging fMRI series can be evaluated, looking for the topological hallmark of the presence of a fourth dimension. Indeed, there is a typical feature which reveal the existence of a functional hypersphere: the simultaneous activation of areas opposite each other on the 3D cortical surface. Our suggestion - substantiated by recent findings - that brain activity takes place on a closed, donut-like trajectory helps to solve long-standing mysteries concerning our psychological activities, such as mind-wandering, memory retrieval, consciousness and dreaming state.

Further proofs of hidden nervous dimensions. Introducing 4D maximal nucleus cluster from algebraic topology of digital images[11]. We introduce a novel method for the measurement of information level in fMRI (functional Magnetic Resonance Imaging) neural data sets, based on image subdivision in small polygons equipped with different entropic content. We show how this method, called maximal nucleus clustering (MNC), is a novel, fast and inexpensive image-analysis technique, independent from the standard blood-oxygen-level dependent signals. MNC facilitates the objective detection of hidden temporal patterns of entropy/information in zones of fMRI images generally not taken into account by the subjective standpoint of the observer. This approach befits the geometric character of fMRIs. The main purpose of this study is to provide a computable framework for fMRI that not only facilitates analyses, but also provides an easily decipherable visualization of structures. This framework commands attention because it is easily implemented using conventional software systems. In order to evaluate the potential applications of MNC, we looked for the presence of a fourth dimension's distinctive hallmarks in a temporal sequence of 2D images taken during spontaneous brain activity. Indeed, recent findings suggest that several brain activities, such as mind-wandering and memory retrieval, might take place in the functional space of a four-dimensional hypersphere, which is a double donut-like structure undetectable in the usual three dimensions. We found that the Rényi entropy is higher in MNC areas than in the surrounding ones, and that these temporal patterns closely resemble the trajectories predicted by the possible presence of a hypersphere in the brain.

Can the nervous connectome be embedeed in hyperbolic spaces[12]? Recent work has shown that the appropriate isometric space for embedding complex

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Multidimensional phase spaces, Simplicial complexes, Networks; torus, Clifford torus; Symmetries, Symmetry breaks; neuroscience, multidimensional brain, Four-dimensional computer, Four-dimensional brain, Connectome, Perceptual spaces, Emotions, Consciousness, Neurological diseases; Top-down rationalism, realism

[10] *Tozzi A, Peters JF. 2016. Towards a Fourth Spatial Dimension of Brain Activity. Cognitive Neurodynamics 10 (3): 189–199. doi:10.1007/s11571-016-9379-z.*

Hypersphere, Clifford torus, Quaternionic movements, Re-BUT; neuroscience, multidimensional brain, resting state, Default mode network, Donut-like brain, Four-dimensional brain

[11] *Peters JF, Ramanna S, Tozzi A, İnan E. 2017. Bold-Independent Computational Entropy Assesses Functional Donut-Like Structures in Brain fMRI Images. Front Hum Neurosci. 2017 Feb 1;11:38. doi: 10.3389/fnhum.2017.00038. eCollection 2017.*

Gradient-based Voronoi tiling, Clifford torus, hypersphere, Computational proximity, MNC; Granger causal modeling, Quaternionic movements; neuroscience, multidimensional brain, Four-dimensional brain, BOLD independent entropy, mind-wandering, spontaneous activity of the brain; entropy as an objective finding

[12] *Tozzi A. 2019. Embeddings of connectome graphs in hyperbolic spaces. (electronic response to: Revealing the Hippocampal Connectome through Super-Resolution 1150-Direction Diffusion MRI. JMaller JJ, Welton T, Middione M, Callaghan FM, Rosenfeld JV, Grieve SM. 2019. Scientific Reports, 9: 2418).*

Hyperbolic spaces, power law, complex networks; neuroscience, brain,

networks (and in particular the neural multidimensional ones, such as the human connectome) is not the flat Euclidean space, but a negatively curved hyperbolic space. Indeed, hyperbolic space has the property that power-law degree distributions, strong clustering and hierarchical community structure emerge naturally when random graphs are embedded in hyperbolic space. It is therefore logical to exploit the structure of the hyperbolic space for useful embeddings of complex networks. It has been demonstrated that, when applied to the task of classifying vertices of complex networks, hyperbolic space embeddings significantly outperform embeddings in Euclidean space.

Mirzakhani's hyperbolic spaces are also in the brain[13]. Biological activities, including cellular metabolic pathways, protein folding and brain function, can be described in terms of curved trajectories in hyperbolic spaces which are constrained by energetic requirements. Here, starting from the recently developed theorems by a deceased Field Medal young mathematician, we show how it is feasible to find and quantify the shortest, energy-sparing functional trajectories taking place in nervous systems' concave phase spaces extracted from real EEG traces. This allows neuroscientists to focus their studies on the few, most prominent functional EEG's paths and loops able to explain, elucidate and experimentally assess the rather elusive mental activity.

The concave phase spaces of the multidimensional brain: response to a Nobel Prize on nervous additional dimensions[14]. Bellmund et al. (2018) define a concept as "a set of CONVEX (i.e., positive curvature) regions of similar stimuli". Such regions might also display other types of curvatures, such as CONCAVE ones. Indeed, several studies point towards many biological and physical dynamics taking place in negative-curvature phase spaces: this is because trajectories on hyperbolic manifolds allow a more manageable treatment of many of the required equations, such as, e.g., the Fokker-Plank ones. Further, parallel transport from Euclidean spaces to concave manifold allows the assessment of nervous multidimensional dynamics in terms of symmetry breaks, and the latter, i.e., a successful approach borrowed from physics, would be very useful in the description and categorization of higher-dimensional manifolds. Linked to the issue of the multidimensional brain and nervous symmetries, stands the fundamental question raised by the Authors: "how a continuous code can be extended to map additional dimensions"? In order to answer, the "evidence of topological representations of spaces in rodents and humans" paves the way to the use of an algebraic topological tool, i.e., the BUT: provided a function is continuous (in this case, "spatially specific cells provide a continuous code"), a single feature in one dimension (say, a sports car) maps to two features with matching description in a dimension higher (two sports cars, which might be slightly different, e.g., in their emotional, or cognitive content). In other words, when I see a cat in my surrounding 3D environment, I perceive not just the 3D image of the real cat in front of me, but also many multidimensional features of the cat in my mind (emotional: "how tender!", cognitive: "this is a Feline", and so on). Therefore, the use of BUT allows us to build symmetric, higher-dimensional topological spaces where mental activity might take place, and to calculate their thermodynamic constraints (given the link between symmetries, informational entropies and topological manifolds).

A technical problem correlated with the curse of dimensionality, i.e., when the spherical dimension increase… the volume decreases[15]! How to avoid the curse of dimensionality, when assessing multidimensional phase spaces? Apart from the canonical techniques used to achieve the "blessing of dimensionality", another, novel approach is available: the Borsuk-Ulam theorem, which has been already widely used in physics, biology and neuroscience.

How to build a 4D computer, ALIAS how to simulate a multidimensional bain: two approaches[16].
1) Through quaternionic networks: the nervous activity of the brain takes place in higher-dimensional

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13 *Tozzi A, Peters JF, Jaušovec N. 2018. EEG dynamics on hyperbolic manifolds. Neurosci Lett, 683: 138-143. https://doi.org/10.1016/j.neulet.2018.07.035.*
Mirzakhani, Hyperbolic surfaces, Polynomial responses; physics, Hilbert space, Riemannian manifolds, goedetics; shortest paths, Langevin equations; neuroscience, brain, hyperbolic EEG

14 *Tozzi A. 2018. How a continuous code can be extended to map additional dimensions? (electronic response to: Bellmund JLS, Gardenfors P, Moser EI, Doeller CF. Navigating cognition: spatial codes for human thinking. Science, 362 (6415), eeat6766).*
Continuous function, parallel transport, topological manifolds, BUT; Concave and convex curvatures, information entropy; neuroscience, multidimensional brain, Four-dimensional brain, Cognitive hippocampus, 4D imagined cat; neuroscience, multidimensional brain,

15 *Tozzi A, Peters JF. 2019. The Borsuk-Ulam theorem solves the curse of dimensionality: Comment on "the unreasonable effectiveness of small neural ensembles in high-dimensional brain" by Alexander N. Gorban et al. Physics of Life Reviews.*
hyperspheres volumes, energy-BUT; Curse of dimensionality, Blessing of dimensionality

16 **Through the Quantum Hall effect:** *Tozzi A, Ahmad MZ, Peters JF. 2020. Neural computing in four spatial dimensions. Cognitive Neurodynamics.* **https://doi.org/10.1007/s11571-020-09598-2.**

Through quaternionic networks: Tozzi A, Peters JF, Jausovec N, Don APH, Ramanna S, Legchenkova I, Bormashenko E. 2021. Nervous Activity of the Brain in Five Dimensions. Biophysica; 1(1):38-47. https://doi.org/10.3390/biophysica1010004.
Clustering maps, Discriminatory union, Shape maps, quaternion, near set theory, shape maps; physics, topological charge pump, Quantum Hall effect, 4D computer, Quantum computer; qbit; quaternions; Multidimensional world neuroscience, multidimensional brain

functional spaces. It has been proposed that the brain might be equipped with phase spaces characterized by four spatial dimensions plus time, instead of the classical three plus time. This suggests that global visualization methods for exploiting four-dimensional maps of three-dimensional experimental data sets might be used in neuroscience. We asked whether it is feasible to describe the four-dimensional trajectories (plus time) of two-dimensional (plus time) electroencephalographic traces (EEG). We made use of quaternion orthographic projections to map to the surface of four-dimensional hyperspheres EEG signal patches treated with Fourier analysis. Once achieved the proper quaternion maps, we show that this multi-dimensional procedure brings undoubted benefits. The treatment of EEG traces with Fourier analysis allows the investigation the scale free activity of the brain in terms of trajectories on hyperspheres and quaternionic networks. Repetitive spatial and temporal patterns undetectable in three dimensions (plus time) are easily enlightened in four dimensions (plus time). Further, a quaternionic approach makes it feasible to identify spatially far apart and temporally distant periodic trajectories with the same features, such as, e.g., the same oscillatory frequency or amplitude. This leads to an incisive operational assessment of global or broken symmetries, domains of attraction inside three-dimensional projections and matching descriptions between the apparently random paths hidden in the very structure of nervous fractal signals.

2) Through the Quantum Hall effect: relationships among near set theory, shape maps and recent accounts of the Quantum Hall effect pave the way to neural networks computations performed in higher dimensions. We illustrate the operational procedure to build a real or artificial neural network able to detect, assess and quantify a fourth spatial dimension. We show how, starting from two-dimensional shapes embedded in a 2D topological charge pump, it is feasible to achieve the corresponding four-dimensional shapes, which encompass a larger amount of information. Synthesis of surface shape components, viewed topologically as shape descriptions in the form of feature vectors that vary over time, leads to a 4D view of cerebral activity. This novel, relatively straightforward architecture permits to increase the amount of available qbits in a fixed volume.

WHAT ARE THE THERMODYNAMIC AND INFORMATION FEATURES OF THE MULTIDIMENSIONAL BRAIN?

Topodynamics of the metastable nervous activity: a survey of the applications of the Borsuk-Ulam theorem to the chaotic, nonlinear brain[17]. The brain displays both the anatomical features of a vast amount of interconnected topological mappings as well as the functional features of a nonlinear, metastable system at the edge of chaos, equipped with a phase space where mental random walks tend towards lower energetic basins. Nevertheless, with the exception of some advanced neuro-anatomic descriptions and present-day connectomic research, very few studies have been addressing the topological path of a brain embedded or embodied in its external and internal environment. Herein, by using new formal tools derived from algebraic topology, we provide an account of the metastable brain, based on the neuro-scientific model of Operational Architectonics of brain-mind functioning. We introduce a "topodynamic" description that shows how the relationships among the countless intertwined spatio-temporal levels of brain functioning can be assessed in terms of projections and mappings that take place on abstract structures, equipped with different dimensions, curvatures and energetic constraints. Such a topodynamical approach, apart from providing a biologically plausible model of brain function that can be operationalized, is also able to tackle the issue of a long-standing dichotomy: it throws indeed a bridge between the subjective, immediate datum of the naïve complex of sensations and mentations and the objective, quantitative, data extracted from experimental neuro-scientific procedures. Importantly, it opens the door to a series of new predictions and future directions of advancement for neuroscientific research.

Neural thermodynamics and information: a topological account of the brain entropies via the Borsuk-Ulam Theorem[18]. Recent approaches to brain phase spaces reinforce the foremost role of symmetries and energy requirements in the assessment of nervous activity. Changes in thermodynamic parameters and dimensions occur in the brain during symmetry breakings and transitions from one functional state to another. Based on topological results and string-like trajectories into nervous energy landscapes, we provide a novel

[17] *Tozzi A, Peters JF, Fingelkurts AA, Fingelkurts AA, Marijuán PC. 2017. Topodynamics of metastable brains. Physics of Life Reviews, 21, 1-20. http://dx.doi.org/10.1016/j.plrev.2017.03.001.*
Worldsheet torus, Affine connections vs causality, Continuous function, All the BUT variants; physics, Kullback-Leibler divergence, Dimensions: definition, Ergodicity, Renewal events, Metastability, Physical duality, 4D computers; neuroscience, multidimensional brain, Phenomenologic spacetime, Fingelkurts' Operational architectonics; Affine connections vs causality, Grigolini, Fingelkurts, Semantics, Syntax, Isomorphic, Emergence, Triad, phenomenology

[18] *Tozzi A, Peters JF. 2017. From abstract topology to real thermodynamic brain activity. Cognitive Neurodynamics, 11(3) 283–292. Doi:10.1007/s11571-017-9431-7.*
string-BUT, general-BUT, energy-BUT; physics, Symmetry breaks, Thermodynamics, Pairwise entropy, information entropy; neuroscience, multidimensional brain, Chaotic brain, Closed and open brain, BUT activity in brain; Holistic, generative model

method for the evaluation of energetic features and constraints in different brain functional activities. We show how abstract approaches, namely the Borsuk-Ulam theorem and its variants, may display real, energetic physical counterparts. When topology meets the physics of the brain, we arrive at a general model of neuronal activity, in terms of multidimensional manifolds and computational geometry, that has the potential to be operationalized.

Sprott's equations in the brain: a single equation describes linear and nonlinear nervous dynamics[19]. The brain is a system at the edge of chaos equipped with nonlinear dynamics and functional energetic landscapes. However, so far no connection has been found between the electric activities of the brain and the physiological repertoire of behavior. Recent work suggests the integrated nature of information processing in the brain not only via synaptic connectivity, but a wholesome physical organization, which takes the shape of a toroidal particle trajectories, chaotic attractors or standing waves. The characterization of brain activities concerning the type of attractors or the trajectories of the nervous phase space is also missing. Starting from a system governed by differential equations in which a dissipative strange attractor coexists with an invariant conservative torus, we developed a 3D model of brain phase space which has the potential to be operationalized and assessed empirically. We achieved a system displaying either a torus or a strange attractor, depending just on the initial conditions. Further, the system generates a funnel-like attractor equipped with a fractal structure. Changes in three brain phase parameters lead to modifications in the funnel's breadth or in torus/attractor superimposition. We have found that the higher frequencies of evoked activities are more deterministic because the greater funnel breadth reduces the degrees of freedom. Thus, evoked activities are more deterministic. In contrast, the resting state corresponds to lower frequencies and represents greater degrees of freedom, which engender daydreaming, mind wandering and other liberal, often arbitrary mental associations. Our model connects the physiological manifestations of consciousness with the electric activities of the brain and it also powerfully explains the differences in motivation between evoked and resting activities based on energy use. This idea might point to the origin of a large repertoire of brain functions, such as sensations/perceptions, memory and self-generated thoughts.

DOES NERVOUS MULTI-DIMENSIONALITY UNDERLIE SPECIFIC COGNITIVE FUNCTIONS OF THE BRAIN?

Weird nervous relationships between information entropies and power laws: when a two-dimensional shadow encompasses more information that the corresponding three-dimensional object[20]. A two-dimensional shadow may encompass more information than its corresponding three-dimensional object. Indeed, if we rotate the object, we achieve a pool of observed shadows from different angulations, gradients, shapes and variable length contours that make it possible for us to increase our available information. Starting from this simple observation, we show how informational entropies might turn out to be useful in the evaluation of scale-free dynamics in the brain. Indeed, brain activity exhibits a scale-free distribution that leads to the variations in the power law exponent typical of different functional neurophysiological states. Here we show that modifications in scaling slope are associated with variations in Rényi entropy, a generalization of Shannon informational entropy. From a three-dimensional object's perspective, by changing its orientation (standing for the cortical scale-free exponent), we detect different two-dimensional shadows from different perception angles (standing for Rènyi entropy in different brain areas). We show how, starting from known values of Rènyi entropy (easily detectable in brain fMRIs or EEG traces), it is feasible to calculate the scaling slope in a given moment and in a given brain area. Because changes in scale-free cortical dynamics modify brain activity, this issue points towards novel approaches to mind reading and description of the forces required for transcranial stimulation.

The brain increases the complexity of sensations: the cat you are watching is three-dimensional in the environment, but multi-dimensional in your mind[21]. Contrary to common belief, the brain appears to increase the complexity from the perceived object to the idea of it. Topological models predict indeed that: a) increases in anatomical/functional dimensions and symmetries occur in the transitionfrom the environment to the higher

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[19] *Peters JF, Tozzi A, Deli E. 2017. Towards Equations for Brain Dynamics and the Concept of Extended Connectome. SF J Neuro Sci 1:1.*
First-order differential equations; hypershphere, torus, phase space; Ohm's law; Sprott equations, Dynamic systems theory, Fixed point attractors, strange attractor, funnel-like, determinism; neuroscience, brain, Extended connectome, consciousness, resting state, chaotic brain, daydreaming, mind wandering, spontaneous activity ogf the brain

[20] *Tozzi A. 2015. Neural code & power laws. SCTPLS Newsletter, April, 7-10. Tozzi A, Peters JF, Cankaya MN. 2018. The informational entropy endowed in cortical oscillations. Cognitive Neurodynamics, 12(5), 501-507. DOI: 10.1007/s11571-018-9491-3.*
Randomly generated angles, 2D object information; shadows; geometric angle; physics, Fractals, poer law exponent, Rényi entropy, Shannon entropy; neuroscience, brain, Transcranial stimulation, EEG traces

[21] *Peters JF, Tozzi A, Ramanna S, Inan E. 2017. The human brain from above: an increase in complexity from environmental stimuli to abstractions. Cognitive Neurodynamics,11(4), 391–394. DOI: 10.1007/s11571-0-17-9428-2.*
Nucleous clustering, MNC entropy; neuroscience, multidimensional brain, Hierarchical processing, Dilution sensory inputs, Imagination; Multidimensional brain, Observer's active role, semantics

activities of the brain, and b) informational entropy in the primary sensory areas is lower than in the higher associative ones. To demonstrate this novel hypothesis, we introduce a straightforward approach to measuring island information levels in fMRI neuroimages, via Rényi entropy derived from tessellated fMRI images. This approach facilitates objective detection of entropy and corresponding information levels in zones of fMRI images generally not taken into account. We found that the Rényi entropy is higher in associative cortices than in the visual primary ones. This suggests that the brain lies in dimensions higher than the environment and that it does not concentrate, but rather dilutes messages coming from external inputs.

Multisensory integration & Borsuk-Ulam theorem: a Machian melting of visual and auditory cues[22]. Recent advances in neuronal multisensory integration suggest that the five senses do not exist in isolation of each other. Perception, cognition and action are integrated at very early levels of central processing, in a densely-coupled system equipped with multisensory interactions occurring at all temporal and spatial stages. In such a novel framework, a concept from the far-flung branch of topology, namely the Borsuk-Ulam theorem, comes into play. The theorem states that when two opposite points on a sphere are projected onto a circumference, they give rise to a single point containing their matching description. Here we show that the theorem applies also to multisensory integration: two environmental stimuli from different sensory modalities display similar features when mapped into cortical neurons. Topological tools not only shed new light on questions concerning the functional architecture of mind and the nature of mental states, but also provide an empirically assessable methodology. We argue that the Borsuk-Ulam theorem is a general principle underlying nervous multisensory integration, resulting in a framework that has the potential to be operationalized. Going beyond Ernst Mach, we suggest the occurrence of complexes of multi-sensations.

Evolutional architecture of multidimensional sensations: the winner takes all in nervous hidden dimensions[23]. A novel demon-based architecture is introduced to elucidate brain functions such as pattern recognition during human perception and mental interpretation of visual scenes. Starting from the topological concepts of invariance and persistence, we introduce a Selfridge pandemonium variant of brain activity that takes into account a novel feature, namely, demons that recognize short straight-line segments, curved lines and scene shapes, such as shape interior, density and texture. Low-level representations of objects can be mapped to higher-level views (our mental interpretations): a series of transformations can be gradually applied to a pattern in a visual scene, without affecting its invariant properties. This makes it possible to construct a symbolic multi-dimensional representation of the environment. These representations can be projected continuously to an object that we have seen and continue to see, thanks to the mapping from shapes in our memory to shapes in Euclidean space. Although perceived shapes are 3-dimensional (plus time), the evaluation of shape features (volume, colour, contour, closeness, texture, and so on) leads to n-dimensional brain landscapes. Here we discuss the advantages of our parallel, hierarchical model in pattern recognition, computer vision and biological nervous system's evolution.

A neuroscientific account of (lower-dimensional) syntax and (higher-dimensional) semantics confirms that the Wittgenstein's Tractatus was right[24]. The discrepancy between syntax and semantics is a painstaking issue that hinders a better comprehension of the underlying neuronal processes in the human brain. In order to tackle the issue, we at first describe a striking correlation between Wittgenstein's Tractatus, that assesses the syntactic relationships between language and world, and Perlovsky's joint language-cognitive computational model, that assesses the semantic relationships between emotions and "knowledge instinct". Once established a correlation between a purely logical approach to the language and computable psychological activities, we aim to find the neural correlates of syntax and semantics in the human brain. Starting from topological arguments, we suggest that the semantic properties of a proposition are processed in higher brain's functional dimensions than the syntactic ones. In a fully reversible process, the syntactic elements embedded in Broca's area project into multiple scattered semantic cortical zones. The presence of higher functional dimensions gives rise to the increase in informational content that takes place in semantic expressions. Therefore, diverse features of human

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22 *Tozzi A, Peters JF. 2017. A Symmetric Approach Elucidates Multisensory Information Integration. Information 8,1. doi: 10.3390/info8010004.*

Re-BUT, BUT as general principle; Entropy values; neuroscience, multidimensional brain, Multisensory neurons, Heteromodal perception, Hierarchical brain; Metaphor, Langacker, Mach, Leonardo da Vinci, Godel, Gardenfors, Kim's emergence, Holdobbler, Peirce, Goodman

23 *Tozzi A, Peters JF. 2018. Multidimensional brain activity dictated by winner-take-all mechanisms. Neuroscience Letters, 678 (21):83-89. https://doi.org/10.1016/j.neulet.2018.05.014.*

Visual images, Pandemonium-BUT; neuroscience, brain, Brain primacy, Perception persistence, Sensation, Neural Darwinism, Four-dimensional brain, Entire perceived object; Connexion vs causality, Gibson, Avenarius, Selfridge, Mach's compex of sensations

24 *Tozzi A, Peters JF, Fingelkurts A, Fingelkurts A, Perlovsky L. 2018. Syntax meets semantics during brain logical computations. ProgrBiophys Mol Biol, 140: 133-141. https://doi.org/10.1016/j.pbiomolbio.2018.05.010.*

Wired friend theorem, String-BUT, four-dimensional computers; Worldsheet, Computer science; neuroscience, multidimensional brain, Knowledge istinct, Language Semantics; Physics od the mind, Perlovski, Wittgenstein, Semantics, Syntax, Godel, logic

language and cognitive world can be assessed in terms of both the logic armor described by the Tractatus, and the neurocomputational techniques at hand. One of our motivations is to build a neuro-computational framework able to provide a feasible explanation for brain's semantic processing, in preparation for novel computers with nodes built into higher dimensions.

A topologically-framed multidimensional theory of knowledge. Starring: Richard Avenarius, Ernst Mach and their phenomenal account of the "mental"[25]. Experience is a process of awareness and mastery of facts or events, gained through actual observation or second-hand knowledge. Recent findings reinforce the idea that a naturalized epistemological approach is needed to further advance our understanding of the nervous mechanisms underlying experience. This essay is an effort to build a coherent topological-based framework able to elucidate particular aspects of experience, e.g., how it is acquired by a single individual, transmitted to others and collectively stored in form of general ideas. Taking into account concepts from neuroscience, algebraic topology and Richard Avenarius' philosophical analytical approach, we provide a scheme which is cast in an empirically testable fashion. In particular, we emphasize the foremost role of variants of the Borsuk-Ulam theorem, which tells us that, when a pair of opposite (antipodal) points on a sphere are mapped onto a single point in Euclidean space, the projection provides a description of both antipodal points. These antipodes stand for nervous functions and activities of the brain correlated with the mechanisms of acquisition and transmission of experience.

Twin paper: Richard Avenarius' Kritik, an English summary[26]. The French-Swiss Professor in inductive philosophy Richard Avenarius (Paris, 1843- Zurich, 1896) is one of the most neglected and misunderstood philosophers. Chunks of his innovative ideas can be found not only in theories of mind/brain such as Gestalt, phenomenalism, behaviourism, functionalism and cybernetics, autopoiesis, dynamical systems theory, embedded/embodied approaches – but also in current neuroscientific concepts such as nervous transduction, electric spikes, neural code, multisensory integration, free-energy principle. We provide the first chronological English summary of his masterpiece, the German-written "Kritik". If he were alive today, I would have not agreed with such an oversimplification: however, we hope he will forgive us, since a better comprehension of his ideas is going to make him a favour....

IS IT FEASIBLE TO DRAW AN ANTI-UNITARY VIEW OF THE BRAIN ACTIVITY?

Supramolecular phrenology: not just spikes! The multifaceted sources of the brain messages[27]! How does central nervous system process information? Current theories are based on two tenets: a) Information is transmitted by action potentials, the language by which neurons communicate with each other – and b) homogeneous neuronal assemblies of cortical circuits operate on these neuronal messages where the operations are characterized by the intrinsic connectivity among neuronal populations. In this view, the size and time course of any spike is stereotypal and the information is restricted to the temporal sequence of the spikes; namely, the "neural code". However, an increasing amount of novel data point towards an alternative hypothesis: a) The role of neural code in information processing is overemphasized. Instead of simply passing messages, action potentials play a role in dynamic coordination at multiple spatial and temporal scales, establishing network interactions across several levels of a hierarchical modular architecture, modulating and regulating the propagation of neuronal messages. b) Information is processed at all levels of neuronal infrastructure from macromolecules to population dynamics. For example, intra-neuronal (changes in protein conformation, concentration and synthesis) and extra-neuronal factors (extracellular proteolysis, substrate patterning, myelin plasticity, microbes, metabolic status) can have a profound effect on neuronal computations. This means molecular message passing may have cognitive connotations. This essay introduces

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25 *Tozzi A, Peters JF. 2017. Towards Topological Mechanisms Underlying Experience Acquisition and Transmission in the Human Brain. Integr Psychol Behav Sci. 51(2), 303–323. doi: 10.1007/s12124-017-9380-z.*
Closeness, Sameness; physics,System, Symmetry, Causality, Alpha-geometric angle, Sphere packing, Dynamic systems theory; neuroscience, multidimensional brain, Free energy principle, Experience, Brain oscillations (spikes), Vital train, Habituation, Consciousness, Characters and elements, Topological perception; Against causality, Old logic, Gibson, Marr, Churchland, Lewin, Whitehead, Godel, Avenarius, Dabaghian, Quine's naturalized epistemology

26 *Tozzi, A. 2018. Richard Avenarius' "Kritik Der ReinenErfahrung" (Critique of Pure Experience): An English Summary. Preprints, 2018090403. doi: 10.20944/preprints201809.0403.v1.*
biomechanics, entropy/energy theories Oscillations, Entropy; Knowledge mechanisms, Evolutionary brain, *cultural evolutionism, linguistics*; *nervous transduction, electric spikes, neural code, multisensory integration, free-energy principle* Avenarius, empiriocriticism, Mach, Gestalt, phenomenalism, behaviourism, functionalism and cybernetics, autopoiesis, dynamical systems theory, embedded/embodied approaches, experimental psychology

27 *Tozzi A. 2015. Information Processing in the CNS: A Supramolecular Chemistry? Cognitive Neurodynamics 9 (5): 463–477.*
Dynamic systems theory, Information storage; neuroscience, brain, Novel receptors, Heartbeats, Extracellular structures, Memory, Neural code, Supramolecular chemistry, protein conformation, concentration and synthesis, extracellular proteolysis, substrate patterning, myelin plasticity, microbes, metabolic status, microcircuitry; Gall, Spurzheim, Jacobs' cones, Friston

the concept of "supramolecular chemistry", involving the storage of information at the molecular level and its retrieval, transfer and processing at the supramolecular level, through transitory non-covalent molecular processes that are self-organized, self-assembled and dynamic. Finally, we note that the cortex comprises extremely heterogeneous cells, with distinct regional variations, macromolecular assembly, receptor repertoire and intrinsic microcircuitry. This suggests that every neuron (or group of neurons) embodies different molecular information that hands an operational effect on neuronal computation.

Voronoi tessellations in histological brain tissues: every neuron is different from another[28]. We provide a novel, fast and cheap method for the morphological evaluation of simple 2-D images taken from histological samples. This method, based on computational geometry, leads to a novel kind of "tessellation" of every type of biological picture, in order to locate the zones equipped with the highest functional activity. As an example, we apply the technique to the evaluation of histological images from brain sections and demonstrate that the cortical layers, rather than being a canonical assembly of homogeneous cells as usually believed, display scattered neuronal micro-clusters equipped with higher activity than the surrounding ones.

IS IT FEASIBLE TO DRAW AN UNITARY VIEW OF THE BRAIN ACTIVITY?

The duality of the brain functions: towards a single nervous activity encompassing all the mental faculties[29]. The term "brain activity" refers to a wide range of mental faculties that can be assessed either on anatomical/functional micro-, meso- and macro-spatiotemporal scales of observation, or on intertwined cortical levels with mutual interactions. Our aim is to show that every brain activity encompassed in a given anatomical or functional level necessarily displays a counterpart in others, i.e., they are "dual". "Duality" refers to the case where two seemingly different physical systems turn out to be equivalent. We describe a method, based on novel topological findings, that makes different manifolds (standing for different brain activities) able to scatter, collide and combine, in order that they merge, condense and stitch together in a quantifiable way. We develop a computational tool which explains how, despite their local cortical functional differences, all mental processes, from perception to emotions, from cognition to mind wandering, may be reduced to a single, general brain activity that takes place in dimensions higher than the classical three-dimensional plus time. This framework permits a topological duality among different brain activities and neuro-techniques, because it holds for all the types of spatio-temporal nervous functions, independent of their cortical location, inter- and intra-level relationships, strength, magnitude and boundaries.

The hairy ball theorem and the electric wave fronts in the brain[30]. Whenever one attempts to comb a hairy ball flat, there will always be at least one tuft of hair at one point on the ball. This seemingly worthless sentence is an informal description of the hairy ball theorem, an invaluable mathematical weapon that has been proven useful to describe a variety of physical/biological processes/phenomena in terms of topology, rather than classical cause/effect relationships. In this paper we will focus on the electrical brain field—electroencephalogram (EEG). As a starting point we consider the recently-raised observation that, when electromagnetic oscillations propagate with a spherical wave front, there must be at least one point of the tangential components of the vector fields where the electromagnetic field vanishes. We show how this description holds also for the electric waves produced by the brain and detectable by EEG. Once located these zero-points in EEG traces, we confirm that they are able to modify the electric wave fronts detectable in the brain. This sheds new light on the functional features of a nonlinear, metastable nervous system at the edge of chaos, based on the neuroscientific model of Operational Architectonics of brain-mind functioning. As an example of practical application of this theorem, we provide testable previsions, suggesting the proper location of transcranial magnetic stimulation's coils to improve the clinical outcomes of drug-resistant epilepsy.

Minimum frustration principle and nervous networks: when evolution dictates our thoughts[31].

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[28] *Peters JF, Tozzi A. Ramanna S. 2016. Brain Tissue Tessellation Shows Absence of Canonical Microcircuits. Neuroscience Letters 626: 99–105. doi:10.1016/j.neulet.2016.03.052.*
Voronoi tessellation, MNC; homogeneity; neuroscience, brain, Chlorocebus Aethiops, Non-homogeneous cortex, Supramolecular phrenology; Morphology and function

[29] *Tozzi A, Peters JF. 2019. The common features of different brain activities. Neurosci Lett, 692 (23): 41-46.* **https://doi.org/10.1016/j.neulet.2018.10.054**.
Elldesbrunner-Hearer nerve, Homotopically equivalent shapes, Wired-friend theorem, Brouwer fixed-point theorem, string-BUT, physics, Dual theories, Branes; neuroscience, brain, Cognition, Emotion/cognition duality; Brain/mind relationship, philosophical schools, Reductionism, Gazzaniga, Locke, De Cusa, Bruno, Holism, Monism

[30] *Tozzi A, Bormashenko E, Jausovec N. 2021. Topology of EEG wave fronts. Cognit Neurodyn. https://doi.org/10.1007/s11571-021-09668-z*
Hairy ball theorem, zero point, zero-point, rigid ball, turus, Uhlenbeck singularity, physics, EEG, wave propagation, antennas, front propagation, electric wave, dandelion plant, Fingelkirts, operational architecture, operational modules, transcranial magnetic stimulation.

[31] *Tozzi A, Fla Tor, Peters JF. 2016. Building a minimum frustration framework for brain functions in long timescales. J Neurosci Res.*94(8): 702–716.
Tozzi A. 2020. Brain timescales and minimum frustration principle (electronic response to: Kaplan HS, Salazar Thula O, Khoss N, Zimmer M. 2019. Nested Neuronal Dynamics Orchestrate a Behavioral Hierarchy across

The minimum frustration principle is a computational approach which states that, in the long timescales of evolution, proteins' free-energy decreases more than expected by thermodynamic constraints as their aminoacids assume conformations progressively closer to the lowest energetic state. Here we show that this general principle, borrowed from protein folding dynamics, can be fruitfully applied to nervous function too. Highlighting the foremost role of energetic requirements, macromolecular dynamics, and, above all, intertwined timescales in brain activity, the minimum frustration principle elucidates a wide range of mental processes, from sensations to memory retrieval. Brain functions are compared to trajectories which, in long nervous timescales, are attracted towards the low-energy bottom of funnel-like structures characterized both by robustness and plasticity. We discuss how the principle, as derived explicitly from evolution and selection of a funneling structure from microdynamics of contacts, is different from other brain models equipped with energy landscapes, such as the Bayesian and free-energy principle and the Hopfield networks. In sum, we make available a novel approach to brain function cast in a biologically informed fashion, with the potential to be operationalized and assessed empirically.

Plasma-like collisionless nervous trajectories: long-range correlations in the brain[32]. Plasma studies depict collisionless, collective movements of charged particles. In touch with these concepts, originally developed by the far-flung branch of high energy physics, here we evaluate the role of collective behaviors and long-range functional couplings of charged particles in brain dynamics. We build a novel, empirically testable, brain model which takes into account collisionless movements of charged particles in a system, the brain, equipped with oscillations. The model is cast in a mathematical fashion with the potential of being operationalized, because it can be assessed in terms of McKean-Vlasov equations, derived from the classical Vlasov equations for plasma. A plasma-like brain also elucidates cortical phase transitions in the context of a brain at the edge of chaos, describing the required order parameters. In sum, showing how the brain might exhibit plasma-like features, we go through the concept of holistic behavior of nervous functions.

Microcolumn resembles a fullerenic structure, giving rise to a barcode[33]. Artificial neural systems and nervous graph theoretical analysis rely upon the stance that the neural code is embodied in logic circuits, e.g., spatio-temporal sequences of ON/OFF spiking neurons. Nevertheless, this assumption does not fully explain complex brain functions. We show how nervous activity, other than logic circuits, could instead depend on topological transformations and symmetry constraints occurring at the micro-level of the cortical microcolumn, i.e., the embryological, anatomical and functional basic unit of the brain. Tubular microcolumns can be flattened in fullerene-like two-dimensional lattices, equipped with about 80 nodes standing for pyramidal neurons where neural computations take place. We show how the countless possible combinations of activated neurons embedded in the lattice resemble a barcode. Despite the fact that further experimental verification is required in order to validate our claim, different assemblies of firing neurons might have the appearance of diverse codes, each one responsible for a single mental activity. A two-dimensional fullerene-like lattice, grounded on simple topological changes standing for pyramidal neurons' activation, not just displays analogies with the real microcolumn's microcircuitry and the neural connectome, but also the potential for the manufacture of plastic, robust and fast artificial networks in robotic forms of full-fledged neural systems.
Fullerenic structures equipped with Stone-Wales transformations have been successfully utilized in the study of macromolecular assemblies. This approach could be useful in the assessment of issues from a far-flung research area, i.e., neuroscience. The basic morphological and functional unit of the brain, the human microcolumn, is a tubular structure that can be flattened in the guise of a fullerene-like two-dimensional lattice. We describe this procedure to build a fullerene-like microcolumn, in which neuronal firing and electric signal propagation are assessed in terms of topological neural network modifications, instead of the canonical logic circuits. Every node stands for a neuron, where neural computations take place. This means that nervous activity, other than logic circuits, could instead depend on topological transformations and symmetry constraints dictated by Stone-Wales transformations occurring in the upper cortical layers. A two-dimensional fullerene-like lattice not only simulates the real microcolumn's microcircuitry, but also makes it possible to build

Timescales. Neuron. pii: S0896-6273(19)30932-8. doi: 10.1016/j.neuron.2019.10.037).
Hamiltonian, Knot theory; Local quasiequilibrium, Minimum frustration principle, Spin glass, Funnel-like attractors; neuroscience, brain, Hopfield networks, Dewetting transitions, Protein folding, Evolution, Sensations, Evolutionary timescales, Supramolecular interactions, Rabinovich, Thoughts as knot, hierarchy of timescales

[32] *Tozzi A, Peters JF, Deli E. 2018. Towards plasma-like collisionless trajectories in the brain. Neurosci Lett.662:105-109. https://doi.org/10.1016/j.neulet.2017.10.016.*
McKean-Vlasov equations, plasma, high energies physics; physics, Phase parameters, Plasma, Deybe sphere, Collisionless movements; neuroscience, brain, Extracellular flows,Torus-like trajectories, fMRI and BUT; Long-range causality

[33] *Tozzi A, Peters JF, Ori O. 2017. Fullerenic-topological tools for honeycomb nanomechanics. Towards a fullerenic approach to brain functions. Fullerenes, Nanotubes and Carbon nanostructures.25 (4): 282-288. http://dx.doi.org/10.1080/1536383X.2017.1283618.*
Tozzi A, Peters JF, Ori O. 2017. Cracking the barcode of fullerene-like cortical microcolumns. Neurosci Letters,644, 100–106. http://dx.doi.org/10.1016/j.neulet.2017.02.064.
Barcodes, Topological defects, Bijective-Re-BUT, hexagonal lattice; physics, Stone-Whales transformations, Fullerenes; neuroscience, brain, microcolumns, exagonal grids, microcircuitry

artificial networks equipped with robustness, plasticity and fastness. Electric signal propagation is investigated in terms of pure topological modifications of the neural honeycomb network.

The Bloch theorem correlates high- and low- brain frequencies[34]. Brain electric activity exhibits two important features: oscillations with different timescales, characterized by diverse functional and psychological outcomes, and a temporal power law distribution. In order to further investigate the relationships between low- and high- frequency spikes in the brain, we used a variant of the Borsuk-Ulam theorem which states that, when we assess the nervous activity as embedded in a sphere equipped with a fractal dimension, we achieve two antipodal points with similar features (the slow and fast, scale-free oscillations). We demonstrate that slow and fast nervous oscillations mirror each other over time via a sinusoid relationship and provide, through the Bloch theorem from solid-state physics, the possible equation which links the two timescale activities. We show that, based on topological findings, nervous activities occurring in micro-levels are projected to single activities at meso- and macro-levels. This means that brain functions assessed at the higher scale of the whole brain necessarily display a counterpart in the lower ones, and vice versa. Our topological approach makes it possible to assess brain functions both based on entropy, and in the general terms of particle trajectories taking place on donut-like manifolds. Condensed brain activities might give rise to ideas and concepts by combination of different functional and anatomical levels. Furthermore, cognitive phenomena, as well as social activity can be described by the laws of quantum mechanics; memories and decisions exhibit holographic organization. In physics, the term duality refers to a case where two seemingly different systems turn out to be equivalent. This topological duality holds for all the types of spatio-temporal brain activities, independent of their inter- and intra-level relationships, strength, magnitude and boundaries, allowing us to connect the physiological manifestations of consciousness to the electric activities of the brain.

The unexpected occurrence of J-functions & complex functions in the brain[35]. The modular function j, central in the assessment of abstract mathematical problems, describes elliptic, intertwined trajectories that move in the planes of both real and complex numbers. Recent clues suggest that the j-function might display a physical counterpart, equipped with a quantifiable real component and a hidden imaginary one, currently undetectable by our senses and instruments. Here we evaluate whether the real part of the modular function can be spotted in the electric activity of the human brain. We assessed EEGs from five healthy males, eyes-closed and resting state, and superimposed the electric traces with the bidimensional curves predicted by the j-function. We found that the two trajectories matched in more than 85% of cases, independent from the subtending electric rhythm and the electrode location. Therefore, the real part of the j-function's peculiar wave is ubiquitously endowed all over normal EEGs paths. We discuss the implications of such correlation in neuroscience and neurology, highlighting how the j-function might stand for the one of the basic oscillations of the brain, and how the still unexplored imaginary part might underlie several physiological and pathological nervous features.

Elliptic curves in the brain: a conceptual revolution[36]? The Diffusion Tensor Imaging and Diffusion Tensor Tractography of neural projections performed by Alizadeh et al. are elliptic curves, i.e., they can be abstractly described in terms of two-dimensional paths without cusps or intersections. These kind of cubic equations' curves are embedded in an algebraic two-dimensional finite field, accurately defined and quantified in terms of points and numbers (both integers and rational). The same type of elliptic curve can be found when examining the wavefronts of EEG and fMRI patterns. What elliptic curves bring on the table, when assessing of brain functions? In our case, elliptic curves (standing for anatomical neural projections detected by tractographic techniques) lie inside a finite field (the brain) which can be subdivided in numbered zones (characterized by integer and rational numbers) and assessed through algebraic weapons, number theory, complex analysis, algebraic geometry and representation theory. Here we provide a few examples. Elliptic curves are equipped with symmetries (they are abelian, in technical terms), apparently hidden at first sight. This allows to compare anatomical/functional neuronal features with matching description which are located in far-flung brain areas. Further, it is noteworthy that half of the elliptic curves displays a low amount of rational numbers, while the other half an infinite number. In our operational terms, this means that half of the nervous patterns are continuous, while half are discontinuous and

[34] *Deli E. Tozzi A, Peters JF. 2017. Relationships between short and fast brain timescales. Cognitive Neurodynamics,* 11(6), 539-552.DOI: 10.1007/s11571-017-9450-4.
Antipodal-BUT, fractal-BUT; physics, Bloch wave, Floquet theorem, Planck-Einstein relation, Holographic principle; neuroscience, brain, Quantum brain

[35] *Tozzi A, Peters JF, Jausovec N. 2016. A repetitive modular oscillation underlies human brain electric activity. Neurosci Lett, 653, 234-238. 10.1016/j.neulet.2017.05.051.*
Modular function, J.function, Compex plane, physics, Monstrous Moonshine; neuroscience, brain, EEG complexity, real and imaginary parts of j-function

[36] *Tozzi A. 2019. Elliptic curves in the central nervous system. (Electronic response to: Alizadeh M, Kozlowski L, Muller J, Ashraf N, ShahrampourS,et al. 2019. Hemispheric Regional Based Analysis of Diffusion Tensor Imaging and Diffusion Tensor Tractography in Patients with Temporal Lobe Epilepsy and Correlation with Patient outcomes. Scientific Reports, 9: 215)*
Elliptic curves, Continuity and discontinuity; torus; neuroscience, brain, Half or neuronal patterns=discontinuous, Connectome in hyperbolic spaces; Diffusion tensor imaging, tractography; continuum

proceed in temporal/spatial quantized steps. The last, but not the least, elliptic curve is a a type of cubic curve whose solutions are confined to a region of space that is topologically equivalent to a torus. This means that anatomical and functional nervous trajectories can be assessed in the easily manageable terms of trajectories on a torus.

Sheaves in your brain: towards an intrinsic mechanism of backpropagation in neuronal networks[37]. Neural networks are commonly described in terms of feedforward paths coupled with feedback circuits that guarantee backpropagation and message control. Here we, based on mathematical arguments from algebraic topology, near set theory and category theory, suggest the testable hypothesis that neuronal networks might be equipped with feedback mechanisms provided by the same feedforward fibers. Once a wheat sheaf is sealed and tied up, the packed down straws display the same orientation: this trivial observation brings us into the realm of presheaves/globular sets, that allow simple assessment of diverging and superimposing functions. We provide a mathematically founded evaluation of nervous activities in terms of presheaves that: a) explain spontaneous random spikes synchronization and b) lead to the counterintuitive insight that neuronal spikes could display antidromic effects, such that, e.g., when an entrained oscillation propagates from A to B, changes in B lead to changes in A. Hierarchical information transmission inside globular sets at various coarse-grained levels of observation provides fresh insights on neural issues such as object persistence, memory reinforcement despite random noise, Bayesian inferential circuits and multidimensional activity of the brain. We provide testable previsions to confirm the theory of neural presheaves/globular sets and suggest to locate transcranial magnetic stimulation's coils downstream from the area of seizure onset, to improve the clinical outcomes of drug-resistant epilepsy. We advocate that axonal stimulation by external sources backpropagates and alters the neuronal electric oscillatory frequency, suggesting the possibility to build neural networks where message feedback is guaranteed by the same feedforward circuits.

A link between travelling waves of neural activity and front waves in chemical reactions[38]. Travelling waves of neural activity crossing the brain networks at mesoscopic/macroscopic scales have been correlated with different functions, from long-term memory to visual stimuli (Zhang et al., 2018). The question of how travelling waves generate and propagate has attracted the attention of various disciplines. We asked whether wave generation/propagation in recurrent nervous networks can be linked with existing physical/chemical models. Here the unusual Belousov–Zhabotinsky reaction (BZ) comes into play. BZ is a nonlinear, chaotic chemical process generating concentric and intersecting waves. Starting from the observation that the brain electric activity displays analogous chaotic and nonlinear behavior, we investigated whether wave propagation of neural oscillations could be described in terms of BZ features. Using a novel artificial network which generates BZ-like concentric waves, we compared the simulated oscillations with the real oscillations from the human brain detectable in fMRI movies of BOLD resting state. We found that the oscillatory patterns produced by our simulations fully overlapped the oscillatory patterns detected in the brain during spontaneous activity, pointing towards the nervous tissue as crossed by diffusive nonlinear oscillatory patterns. The excellent agreement between measured and simulated data suggests that the patterns of nervous travelling waves are predictable, being the changes in neural activity correlated with the transient occurrence of (either short or long-range) BZ-like oscillatory patterns in cortical areas. Also, we describe benefits, limits and biological plausibility of BZ-like neural networks and suggest that chaotic dynamics might simply arise from the very structure of network arrangements subtending physical/biological systems.

Knots, braids and… anyons in your brain[39]. The dynamical processes of living systems are characterized by the cooperative interaction of many units. This claim enables us to portray the embryo-fetal development of the central and peripheral nervous systems in terms of assemblies of building blocks. We describe how the structure and arrangement of nervous fibers is - at least partially - dictated by biophysical and topological constraints. The far-flung field of soft-matter polymers/nematic colloids sheds new light on the neurulation in mammalian embryos, suggesting an

[37] *Tozzi A. 2019. Sheaves in the Brain Elucidate the Behavior of Entrained Oscillations. Preprints 2019. doi: 10.20944/preprints201912.0179.v1.*
Composition of functions, Sheaf theory, presheaves, sheaves, globular sets, infinity topoi, cohomology groups, Lurie, sheaf cohomology, stalk, infinity category, amplituhedron, neuroscience, brain, diffusion fMRI, backpropagation; feedforward, feedback; anterograde fibers, retrograde, synchronization, entrainment, transcranial magnetic stimulation, Macaca mulatta histology, multidimensional brain.

[38] *Zhang Y, Guo S, Sun M, Tozzi A, Zhao X. 2019. Non-equilibrium Belousov-Zhabotinsky reaction describes the spontaneous activity of the brain. BioRxiv. doi: https://doi.org/10.1101/863324.*

Nonequilibrium thermodynamics, chemical reaction, fMRI Mitra, lag thread, one-bit decoder, multi-bit binary decoder, Belousov-Zhabotinsky reaction

[39] *Tozzi, A. Biophysics of Nervous Embryo-Fetal Development. Preprints 2021, 2021030111. doi: 10.20944/preprints202103.0111.v1.*
knot theory, braid groups, Frank-Kasper phases, quasicrystals assemblies, fullerene-like cortical microcolumns, D'Arcy Thompson

intriguing testable hypothesis: the development of the central and peripheral nervous systems might be correlated with the occurrence of local thermal changes in embryo-fetal tissues. Further, we show a correlation between the fullerene-like arrangement of the cortical microcolumns and the Frank-Kasper phases of artificial quasicrystals assemblies. The last, but not the least, we explain how and why the multisynaptic ascending nervous fibers connecting the peripheral receptors to the neocortical areas can be viewed as the real counterpart of mathematical tools such as knot theory and braid groups. Their group structure and generator operations point towards a novel approach to long-standing questions concerning human sensation and perception, leading to the suggestion that the very arrangement and intermingling of the peripheral nervous fibers contributes to the cortical brain activity. In touch with the old claims of D'Arcy Thompson, we conclude that the arrangement and the pattern make the function in a variety of biological instances, leading to countless testable hypotheses.

SPECIFIC ACTIVITIES OF THE BRAIN: HOW FAR WE CAN GET WITH UNCONVENIONAL APPROACHES?

The spontaneous activity of the brain displays lower information entropy than task-related activities[40]. Spontaneous activity of the brain is generated in the absence of an explicit task and hence frequently associated to resting-state or default-network functions. Despite its recent discovery has shed new light on questions concerning the structural and functional architecture of the brain and how they are related to "mind", several issues still need to be assessed. In this review, we focus on the scarcely explored energetic requirements and constraints of spontaneous activity, taking into account both thermodynamical and informational standpoints. At first, we argue that the "classical" definitions of spontaneous activity do not take into account an important feature. Indeed, spontaneous brain activity is equipped with slower oscillations compared with the evoked, task-related one, hence it exhibits lower levels of enthalpy and free-energy. Therefore, noteworthy thermodynamic energetic differences occur between spontaneous and evoked brain activities. It means that the brain functions traditionally associated with spontaneous activity, such as mind wandering and so on, require less energy that other nervous activities. We also review recent empirical observations in neuroscience, in an attempt to capture how spontaneous brain dynamics and mental function can be embedded in a non-linear dynamical framework, which talks about nervous activity in terms of phase spaces, particle trajectories, random walks, attractors and/or paths at the edge of the chaos. This takes us from the thermodynamic free-energy to the realm of variational free-energy, a theoretical construct pertaining to probability and information theory and able to explain several unexplored features of spontaneous.

Einstein's special relativity and thoughts: why an imagined object is squeezed in jour mind[41]. In touch with the Authors' strong physicalist claims, we take a step further: to encompass the dynamics described by the Queen of physics, i.e., special relativity, into the fruitful framework of dynamic logic. Indeed, the subjective perception of time could be assessed through the objective reference frame described by Einstein's four-dimensional spacetime in special relativity.
When perceived by the human mind, an object might encompass diverse content according to different observers. Further, subjectively experienced time is encoded in the later entorhinal cortex. Starting from these two observations concerning mental perception of space and time, and considering Einstein's accounts, we show how, in terms of special relativity, imagination's content is not stationary and fixed, rather depends on the observer's standpoint. We elucidate how the subjective phenomenon of time (perceived by our mind as static) might give rise to changes in quantifiable content between the real and the imagined object. We describe how to correlate the quantifiable content of the sensed object embedded in the environment with the corresponding internal thought (subjective percept). In particular, based on recent neuroscientific literature, we show how changes in our mental time windows are able to squeeze the information content of the subjective percepts, compared with their matching environmental objects. Further, we elucidate how this novel framework could be able to confirm or reject a recently raised hypothesis, which suggests that the brain activity takes place in functional dimensions higher than our usual four-dimensional spacetime.

Gibson 4.0: the topological horizons of the ecological theory of vision[42]. During the exploration of the

[40] *Tozzi A, Zare M, Benasich AA. 2016. New Perspectives on Spontaneous Brain Activity: Dynamic Networks and Energy Matter. Front Hum Neurosci. 10:247. doi: 10.3389/fnhum.2016.00247.*
physics, Ohm's law, Thermodynamic entropy, enthalpy, Free energy principle, Dynamic systems theory; neuroscience, brain, CNS random walks, Spontaneous activity of the brain, Default mode network; Friston

[41] *Tozzi A. 2018. Einstein and the physics of the mind: Comment on "Physics of mind: experimental confirmations of theoretical predictions" by Felix Schoeller et al.Phys* Life Rev, https://doi.org/10.1016/j.plrev.2018.01.009.
Tozzi A. 2018. When Einstein's relativity meets neuroscience. PsyArXiv, 10.31234/osf.io/3n6c8.
Special relativity; neuroscience, brain, Imagination, Mental image, Information in thoughts, Hypnosis, Dreams, Tunnel experience; Carnap

[42] *Tozzi A, Peters JF. 2019. Topology of Human Perception. Preprints, 2019030235.*
Lusternick-Schnirelmann theorem, Shapes, BUT variants; shape pattern, topological invariance; Symmetries, Ecological

surrounding environment, the brain links together external inputs, giving rise to perception of a persisting object. During imaginative processes, the same object can be recalled in mind even if it is out of sight. Here, topological theory of shape provides a mathematical foundation for the notion of persistence perception. In particular, we focus on ecological theories of perception, that account for our knowledge of world objects by borrowing a concept of invariance in topology. We show how a series of transformations can be gradually applied to a pattern, in particular to the shape of an object, without affecting its invariant properties, such as boundedness of parts of a visual scene. High-level representations of objects in our environment are mapped to simplified views (our interpretations) of the objects, in order to construct a symbolic representation of the environment. The representations can be projected continuously to an environmental object that we have seen and continue to see, thanks to the mapping from shapes in our memory to shapes in Euclidean space.

IS IT FEASIBLE TO USE GAUGE THEORIES FROM PARTICLE PHYSICS TO DESCRIBE BIOPHYSICAL ISSUES?

The brain from above: a gauge theory of nervous activity. Feat. Karl Friston and Biswa Sengupta[43]. Given the amount of knowledge and data accruing in the neurosciences, is it time to formulate a general principle for neuronal dynamics that holds at evolutionary, developmental, and perceptual timescales? In this paper, we propose that the brain (and other self-organized biological systems) can be characterized via the mathematical apparatus of a gauge theory. The picture that emerges from this approach suggests that any biological system (from a neuron to an organism) can be cast as resolving uncertainty about its external milieu, either by changing its internal states or its relationship to the environment. Using formal arguments, we show that a gauge theory for neuronal dynamics—based on approximate Bayesian inference—has the potential to shed new light on phenomena that have thus far eluded a formal description, such as attention and the link between action and perception.

Neural Gauge theory: novel insights in a book chapter (Springer)[44]. Recent advances in neuroscience highlight the complexity of the central nervous system (CNS) and call for general, multidisciplinary theoretical approaches. The aim of this chapter is to assess highly organized biological systems, in particular the CNS, via the physical and mathematical procedures of gauge theory – and to provide quantitative methods for experimental assessment. We first describe the nature of a gauge theory in physics, in a language addressed to an interdisciplinary audience. Then we examine the possibility that brain activity is driven by one or more continuous forces, called gauge fields, originating inside or outside the CNS. In particular, we go through the idea of symmetries, which is the cornerstone of gauge theories, and illustrate examples of possible gauge fields in the CNS. A deeper knowledge of gauge theories may lead to novel approaches to (self) organized biological systems, improve our understanding of brain activity and disease, and pave the way to innovative therapeutic interventions.

The life from above: a gauge theory of cellular activity[45]. Based on novel topological considerations, we postulate a gauge symmetry for living cells and proceed to interpret it from a consistent Eastern perspective: the li organization principle. Gauge theories had a tremendous impact in particle physics and have been recently proposed in order to assess nervous activity too. Herein, taking into account novel claims from topology, the mathematical branch that allows the investigation of the most general systems activity, we aim to sketch a gauge theory addressed to the fundamentals of cellular organization. In our framework, the reference system is the living cell, equipped with general symmetries and energetic constraints standing for the intertwined biochemical, metabolic and signaling

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theories; neuroscience, brain, Perception, Persistence of perception, high-level representations; Gibson, Phenomenology, Mach, continuous persistence, epistemology

[43] *Sengupta B, Tozzi A, Coray GK, Douglas PK, Friston KJ. 2016. Towards a Neuronal Gauge Theory. PLOS Biology 14 (3): e1002400. doi:10.1371/journal.pbio.1002400.*

Invariance properties, Data analysis, Inference scheme, Probability distributions, Generative models, Differential geometry tutorial; Fisher information metric, Levi-Civita connexion, Bayesian; physics, Global transformations, Gauge field, Gradient ascent, Variational free-energy, Stochastic approaches; neuroscience, brain, Hierarchical, Attention,Perception, Action, Prediction errors, Bottom-up; Kant, Friston

[44] *Tozzi A, Sengupta B, Peters JF,Friston KJ. 2017. Gauge Fields in the Central Nervous System.193-212. In: The Physics of the Mind and Brain Disorders: Integrated Neural Circuits Supporting the Emergence of Mind, edited by Opris J and Casanova MF. New York, Springer; Series in Cognitive and Neural Systems.Pages 193-212. ISBN: 978-3-319-29674-6. DOI10.1007/978-3-319-29674-6_9.*

Ehresmann connexion, gauge-BUT; physics, Free-energy principle, Time as gauge field; neuroscience, brain, Scale/free fluctuations, E/I ratio, Microcolumns, Surprise, Consciousness, Ascending arousal system, Blood flow and Heart rate; Angelo Mosso, Thompson D'Arcy, Whitehead, Kurt Lewin, Churchland, Fechner, Autopoiesis (Maturana, Varela)

[45] *Tozzi A, Peters JF, Navarro J, Kun W, Lin B, Marijuán PC. 2017. Cellular Gauge Symmetry and the Li Organization Principle. Progress in Biophysics and Molecular Biology.* **https://doi.org/10.1016/j.pbiomolbio.2017.06.004**.

General-BUT, energy-BUT, string-BUT; Gauge symmetry, Complexity index, Energy rate density, Energetic atomic levels, information; Chausson's complexity, Topological cells, Prokaryotes, Eukaryotes, Biochemical pathways, Cellular homeostasis, Minimal bacterial genome; Zhu Xi, Capra, Wu, Craig Venter, Chausson

pathways that allow the global homeostasis of the system. Abstractly, these functional movements would follow donut-like trajectories. Environmental stimuli stand for forces able to locally break the symmetry of metabolic and signaling pathways, while the species-specific DNA is the gauge field that restores the global homeostasis after external perturbations. We show how the Borsuk-Ulam Theorem (BUT), which states that a single point on a circumference maps two points on a sphere, allows an inquiry on the evolution from inorganic to organic structures as well as the comparison between prokaryotic and eukaryotic metabolisms and modes of organization. Furthermore, using recently developed BUT variants, we operationalize a methodology for the description of cellular activity in terms of topology/gauge fields and discuss about the experimental implications and feasible applications. We converge on the strategic role that second messengers have played regarding the emergence of such a unitary gauge field for the cell, and the subsequent evolutionary implications for multicellulars. A new avenue for a deeper investigation of biological complexity looms. Philosophically, along this overall exploration of cellular dynamics and biological complexity, we might be reminded of the duality between two essential concepts proposed by the great Chinese synthesizer Zhu Xi (in the XIII Century). His explanatory scheme epitomizes a feasible philosophical interpretation of the present proposal: on the one side, the li organization principle, which may be taken as equivalent to the dynamic interplay between symmetry and information; and on the other side, the qi principle, which can be interpreted as the energy participating in the process, and which always appears as interlinked with the former. In contemporary terms, it would mean the required interconnection between information and energy, and at the same time it would be pointing at essential interpretive principles of information philosophy.

A gauge theory for timeless biology: when time does not count[46]. Contrary to claims that physics is timeless while biology is time-dependent, we take the opposite standpoint: physical systems' dynamics are constrained by the arrow of time, while living assemblies are time-independent. Indeed, the concepts of "constraints" and "displacements" shed new light on the role of continuous time flow in life evolution, allowing us to sketch a physical gauge theory for biological systems in long timescales. In the very short timescales of biological systems' individual lives, time looks like "frozen" and "fixed", so that the second law of thermodynamics is momentarily wrecked. The global symmetries (standing for biological constrained trajectories, i.e. the energetic gradient flows dictated by the second law of thermodynamics in long timescales) are broken by local "displacements" where time is held constant, i.e., modifications occurring in living systems. Such displacements stand for brief local forces, able to temporarily "break" the cosmic increase in entropy. The force able to restore the symmetries (called "gauge field") stands for the very long timescales of biological evolution. Therefore, at the very low speeds of life evolution, time is no longer one of the four phase space coordinates of a spacetime Universe: but it becomes just a gauge field superimposed to three-dimensional biological systems. We discuss the implications in biology: when assessing living beings, the underrated role of isolated "spatial" modifications needs to be emphasized, living apart the evolutionary role of time.

IS THE WORLD MULTIDIMENSIONAL?

The Borsuk-Ulam theorem as a general principle for a multidimensional world[47]. Recently introduced versions of the Borsuk-Ulam theorem (BUT) reveal that a feature vector on a n-manifold projects two feature vectors (matching descriptions of a single object) onto an n+1 manifold. Starting from this rather simple, yet far-reaching, computational topology observation, we build a fruitful general framework, able to elucidate disparate "real" physical and biological phenomena, from quantum entanglement to gauge theories. Summarizing this novel topological approach, we take into account projections among functional or real dimensions. We achieve a system of mappings that fit very well with experimental results, making it possible to assess countless systems in far-flung scientific branches. This book highlights the computational character of matching descriptions (arising from descriptively proximal objects) that display a widest range of possible uses. Such observations point to BUT not just from the standpoint of a novel interpretation of almost all the biological and physical phenomena, but also as suitable tools in evaluating the slight (objective and subjective) differences that make our world an astonishing realm of rich heterogeneity.

A novel theorem
Gravitational lensing
Pauli exclusion principle
Small world networks

Hausdorff measure, Group theory, Lodato proximity, Internal-BUT, Three-BUT, BUT as a general principle, mappings, projections; physics, Fourth dimensions, Entropy difference, Renyi entropy, Shannon entropy, Ergodicity, Small world networks, Gravitational lensing, Donut-like structures, Bosons, Pauli principle, computation; Projections and evolution, Origins of species, Dimensions living beings, Protein folding; Projectionism, End of causality, Strogatz, Identity, Heidegger, Multidimensional world

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[46] *Tozzi A, Peters JF, Chafin C, De Falco D, Today J. 2018. A timeless biology. Progr Biophys Mol Biol. 134, 38-43. doi: 10.1016/j.pbiomolbio.2017.12.002.* Virtual constraints, Virtual displacement, Parallax mapping; Time in quantum mechanics, Shannon entropy plus time, Gauge procedures, symmetry; Spatial modifications, life, evolution; Decreasing the role of time in life

[47] *Tozzi A, Peters JF. 2017. The multidimensional world. Lambert Academic Publishing, Saarbrücken, Germany. ISBN-13: 978-3-330-03530-0.*

Ergodicity
Group theory
Thermodynamic entropy: the arrow of time
Shannon, Rényi entropy
The dimensions of living beings
Natural projections and evolution
How to detect hidden dimensions?

Multidimensional flows in the physical world: chaotic four-dimensional paths & the Feigenbaum constant[48]. The possible presence of further dimensions hidden in our three-dimensional-plus time world might help to elucidate countless physical and biological systems' behaviors, from quantum entanglement to brain function. Nevertheless, suggestions concerning multidimensional arrangement of physical and biological systems do not deserve the role of scientific claims, unless the suggested additional dimensions can be verified via empirically testable hypotheses and experimental apparatus. Here we suggest that the widespread nonlinear dynamics and chaotic behavior of physical and biological collective systems might mirror further dimensions hidden in our world. Indeed, bringing together disparate knowledge from seemingly unrelated fields (brane cosmology, fluid dynamics, algebraic topology, computational topology, dynamic systems theory, logic and statistical mechanics), we show how, in logistic maps derived from nonlinear dynamical equations, the typical bifurcation diagrams might arise from linear flow paths, that intersect largesized hidden dimensions at the canonical phase parameter's values between three and four. Therefore, chaotic dynamics suggests the existence of a further hidden dimension in our Universe. We also provide a thermodynamic framework which suggests that the cosmic entropy is encompassed in a multidimensional manifold.

Dynamic systems theory and evolution[49]. Lysenko suggested the heritability of acquired characteristics. His heretic ideas were dismissed with disgust in favor of the "post-Darwinist" standard evolution theory (SET), one of the most pervasive paradigms of the modern science. However, after half a century of oblivion, the debate is once again an hot topic of current research. In particular, the possible epigenetic inheritance within organisms have been suggested as neo-Lamarckian in nature and talks about a picture different from SET, despite Wray's skeptical claims. If we examine the problem from the novel perspective of the supramolecular chemistry, we notice that the epigenetic information involves the storage of information at the molecular level and its retrieval, transfer and processing at the supramolecular level, via transitory processes that are self-organized, self-assembled and dynamic. SET does not keep into account that the complexity of adaptive evolving systems (including species, niches and environment) is best understood as dynamic networks of relationships, aiming to decrease their free energy via entropy transfer. The DNA is just one of the countless functional tasks of interest in the study of evolution: changes propagate through interlinked levels of organization, inducing connectivity and interaction on all scales of the multilevel system, with no preferred level of granularity. Models of fitness attractors intended to capture the process of natural selection are starting to be developed, taking into account power laws, non-equilibrium steady-state at the edge of the chaos and energetic landscapes made of basins, valleys, floors, ridges and saddle points. In conclusion, it would be useful to investigate SET in the framework of dynamical system theories.

Building fractal oscillations from noise[50]. Scale-free dynamics are an intrinsic feature of a large class of natural models, from earthquakes to brain activity. Assessing a geometrical/mathematical model of synthetic power law oscillations, we noticed that a wave containing a fractal-like structure can be produced by summing a random oscillation to a carefully chosen one. This observation gives rise to countless applications: a "hidden" oscillation may cause a scale-free behavior in a random noise; a fractal system can be produced by simply choosing the appropriate oscillation to bring in; if power laws are involved in random walks, phase transitions and self-organized criticality, then the superimposition of a carefully chosen oscillation may lead to systems of increased complexity; "nested" waves from the central nervous system's spontaneous networks may be the source of the scale-free dynamics seen in EEG and fMRI; in the event of brain 1/f scaling disruption caused by illnesses such as Alzheimer's disease, an external wave - for instance, via transcranial stimulation - could restore the broken symmetry.

Geometric curves underlying physical and biological dynamics[51]. We illustrate a recently-developed geometrical physical model able to assess electronic

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48 *Tozzi A, Peters JF, James III C, Deli E. 2016. Multidimensional chaotic dynamics and entropies. Archives of Information Science and Technology, 1(1): 10-19.*
Logistic plots, MNC tessellations, Feigenbaum constant; physics, Beckenstein bound, cosmic entropy, Branes, Fourth dimension, Flows, Zeeman effect, bifurcation plot, phase parameter

49 *Tozzi A. 2014. Evolution: Networks and Energy Count. Nature 515: 343. doi:10.1038/515343c.*
Networks; Dynamic systems theory, Edge of chaos, self-organization; Evolution, acquired traits, epigenetics, supramolecular chemistry, fitness, hierarchical; Lysenko, Lamarck, Darwin

50 *Tozzi A. 2015. How to Turn an Oscillation in a Pink One. Journal of Theoretical Biology 377, 117–18. doi:10.1016/j.jtbi.2015.04.018.*
Sine waves; Fractals, Power laws, scale-free, random noise, random walks, phase transitions, criticality; Transcranial stimulation, fMRI

51 *Yurkin A, Tozzi A, Peters JF, Marijuan PC. 2017. Quantifying Energetic Dynamics in Physical and Biological Systems Through a Simple Geometric Tool and Geodetic Curves. Addendum to: Cellular Gauge Symmetry and the Li Organization Principle. Progress in Biophysics and Molecular Biology. https://doi.org/10.1016/j.pbiomolbio.2017.06.007.*

movements and energetic paths in atomic shells. The model describes a multi-level system of circular, wavy and zigzag paths which can be projected onto a horizontal tape. This model ushers in a visual interpretation of the distribution of atomic electrons' energy levels and the corresponding quantum numbers through rather simple tools, such as compasses, rulers and straightforward calculations. Here we show how this geometrical model, with the due corrections, among them the use of geodetic curves, might be able to describe and quantify the structure and the temporal development of countless physical and biological systems, from Langevin equations for random paths, to symmetry breaks occurring ubiquitously in physical and biological phenomena, to the relationships among different frequencies of EEG electric spikes. Therefore, in our work we explore the possible association of binomial distribution and geodetic curves configuring a uniform approach for the research of natural phenomena, in biology, medicine or the neurosciences.

Time-reversal entropy: an underrated actor[52]. The paper by Ramstead et al. reminds us the efforts of eminent scientists such as Whitehead and Godel. After having produced influential manuscripts, they turned to more philosophical issues, understanding the need for a larger formalization of their bounteous scientific results. In a similar way, the successful free-energy principle has been generalized, in order to encompass not only the brain activity of the original formulation, but also the whole spectrum of life. Here we go through philosophical (the principle of identity) and physical (temperature, Pandemonium architecture, time reversal entropy) issues that might be correlated with the free energy principle.

MAY ANCIENT PHILOSOPHICAL DISPUTES LEAD TO A NOVEL PHYSICS?

Against the Ockham's razor. The Multiple before the One: what if Alain Badiou was right[53]? One of the criteria to a strong principle in natural sciences is simplicity. The conventional view holds that the world is provided with natural laws that must be simple. This common-sense approach is a modern rewording of the medieval philosophical/theological concept of the Multiple arising from (and generated by) the One. Humans need to pursue unifying frameworks, classificatory criteria and theories of everything. Still, the fact that our cognitive abilities tend towards simplification and groupings does not necessarily entail that this is the way the world works. Here we ask: what if singularity does not pave the way to multiplicity? How will we be sure if the Ockham's razor holds in real life? We will show in the sequel that the propensity to reduce to simplicity the relationships among the events leads to misleading interpretations of scientific issues. We are not going to take a full sceptic turn: we will engage in active outreach, suggesting examples from biology and physics to demonstrate how a novel methodological antiunitary approach might help to improve our scientific attitude towards world affairs. We will provide examples from aggregation of SARS-Cov-2 particles, unclassified extinct creatures, pathological brain stiffness. Further, we will describe how antiunitary strategies, plagiarising medieval concepts from William od Ockham and Gregory of Rimini, help to explain novel relational approaches to quantum mechanics and the epistemological role of our mind in building the real world.

Yurkin rays, Grid deformations, Geometric tools, geodetics; quantum energy levels; atomic energetic levels, Langevin equations, symmetry break; Protein folding's timescales, Metastasis, Default mode network, Energy budget, Different EEG frequencies spikes; Wigner

[52] *Tozzi A. Peters JF. 2017. Critique of pure free energy principle: Comment on "Answering Schrödinger's question: A free-energy formulation" by Maxwell James DésormeauRamstead et al. Physics of Life Reviews.DOI: 10.1016/j.plrev.2017.10.003.*

Set theory, Identity, Pandemonium; Free-energy principle, Markov blancket, Time-reversal entropy; neuroscience, Brain temperature, evolution; Whitehead, Godel, Badiou, Bechtel, Helmotz, Heidegger

[53] *Tozzi A. 2021. Why Should Natural Principles Be Simple? Philosophia. https://doi.org/10.1007/s11406-021-00359-x*

quantum field theory, Mittag-Leffler equations, Dickinsonia, COVID children, Golgi, Endoplasmic reticulum; Ockham, relational quantum mechanics, brain stiffness, Rovelli; Mpemba effect, from monocellular to multicellular, viral collective aggregates, SARS-CoV-2 particles, virions; viral clusters, neuroscience, multidimensional brain; McGinn, Hilbert's 24th problem, Ockham's razor, Hermes Trismegistus, Alain Badiou, Deleuze and Guattari, Henry of Harclay, Bechtel's multilevel emergentism. Thomas Aquinas, First Principles, BIG Bell Test Collaboration, Eucharist, Gregory of Rimini and *complexe significabile*, Adam of Wodeham, University of Paris

PLUS: **Dewetting antibodies against Influenza A and Covid-19**

Tozzi A. 2019. Towards Dewetting Monoclonal Antibodies for Therapeutical Purposes. Preprints. doi: 10.20944/preprints201905.0356.v1.

Tozzi A. 2020. BK channels undergo dewetting transitions: therapeutical implications (electronic response to: Sun AX, Yuan Q, Fukuda M, Yu W, Yan H, et al. 2019. Potassium channel dysfunction in human neuronal models of Angelman syndrome. Science, 366 (6472): 1486-1492. DOI: 10.1126/science.aav5386).

Tozzi A. 2020. Dewetting Antibodies Against Viroporins Might Contribute to Decrease 2019-Ncov Infection Spread. viXra:2002.0046

physics, Fluid mechanics, Gas/surface tension, Lifshitz-van Der Wall interactions, Superhydrophobic structures; Extracellular matrix, lipid bilayers, receptors, active sites of channels, nervous ion channels, Escherichia coli, Influenza A's M2; Pseudomonas; Coronavirus, Engelmann syndrome, diabetes.

Induction reloaded: heretic topological approaches to cause/effect relationships[54]. Causal relationships lie at the very core of scientific description of biophysical phenomena. Nevertheless, observable facts involving changes in system shape, dimension and symmetry may elude simple cause and effect inductive explanations. Here we argue that numerous physical and biological phenomena such as chaotic dynamics, symmetry breaking, long-range collisionless neural interactions, zero-valued energy singularities, and particle/wave duality can be accounted for in terms of purely topological mechanisms devoid of causality. We illustrate how simple topological claims, seemingly far away from scientific inquiry (e.g., "given at least some wind on Earth, there must at all times be a cyclone or anticyclone somewhere"; "if one stirs to dissolve a lump of sugar in a cup of coffee, it appears there is always a point without motion"; "at any moment, there is always a pair of antipodal points on the Earth's surface with equal temperatures and barometric pressures") reflect the action of non-causal topological rules. To do so, we introduce some fundamental topological tools and illustrate how phenomena such as double slit experiments, cellular mechanisms and some aspects of brain function can be explained in terms of geometric projections and mappings, rather than local physical effects. We conclude that unavoidable, passive, spontaneous topological modifications may lead to novel functional biophysical features, independent of exerted physical forces, thermodynamic constraints, temporal correlations and probabilistic a priori knowledge of previous cases.

Heidegger's Being & quantum vacuum: Being, Essence, Existence and their physical counterparts55. A dialogue between Martin Heidegger and a theoretical physicist, namely Richie, unveils the striking relationships between the Eastern and Western philosophical concepts of Being and the experimentally detectable quantum vacuum. We provide an account of long-standing theoretical issues, such Being, Entity, Existence and the unique role of the human Thoughts in the world and expound their possible physical counterparts.

"The same" in biology. The principle of identity reloaded: When A=A turns out A≠A[56]. A unifying principle underlies the organization of physical and biological systems. It relates to a well-known topological theorem which succinctly states that an activity on a planar circumference projects to two activities with "matching description" into a sphere. Here we ask: what does "matching description" mean? Has it something to do with "identity"? Going through different formulations of the principle of identity, we describe diverse possible meanings of the term "matching description". We demonstrate that the concepts of "sameness", "equality", "belonging together" stand for intertwined levels with mutual interactions. By showing that "matching" description is a very general and malleable concept, we provide a novel testable approach to "identity" that yields helpful insights into physical and biological matters. Indeed, we illustrate how a novel mathematical approach derived from the Borsuk-Ulam theorem, termed bio-BUT, might explain the astonishing biological "multiplicity from identity" of evolving living beings as well as their biochemical arrangements.

When the principle of non-contradiction fails: Nicholas of Autrecourt anticipated Paraconsistent Logics[57]. We suggest that the 14th century scholar Nicholas of Autrecourt can be regarded as a precursor of the paraconsistent logics developed around 1950. We show how the Sorbonne licentiatus in theology provided in his few extant writings a refutation of both the principle of explosion and the law of non-contradiction, in accordance with the tenets of paraconsistent logics. This paves the way to the most advanced theories of truth in natural language and quantum dynamics.

Debunking Poppers's falsifiability[58]. It has been stated that "a founding principle in science is the ability to

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[54] *Tozzi A, Papo D. 2020. Projective mechanisms subtending real world phenomena wipe away cause effect relationships. Progress in Biophysics and Molecular Biology. 151:1-13. DOI: 10.1016/j.pbiomolbio.2019.12.002.*
Tozzi A. 2019. RE: towards applications for exceptional points also in non-hermitian biological networks. (electronic response to: Miri M-A, Alù A. 2019. Exceptional points in optics and photonics. Science, 363 (6422), eaar7709. DOI: 10.1126/science.aar7709.
Kneser graphs, Patch connected subsets, Ham sandwich theorem, Brower fixed point theorem, Wired friend theorem, Lusternik-Schnirelmann theorem, Hairy ball theorem, Ulenbeck equations, Eigenvalue surfaces, Exceptional points, operational-BUT, BUT as universal principle, Split-BUT; fragile topology, magic angle, superconductor, graphene; physics, Causality, Entropy, Shannon plot, Double slit experiment, Particle/wave duality, Gravitational lensing, Plasma collisionless movements, Ergodicity, II law thermodynamics, Non-Hermitian systems, Cavity-laser sources, information; Adverse drug reactions; Mytosis, Ecological approach to visual perception, Cellular membrane, Dewetting transition, Social processes, Intestinal organoids, Viral immunity, Singularities in CNS, E/I ratio, Imaginary values, Life definition, Inanimate matter's evolution, life origin; Causality, Projectionism, Feddoso, Al-Ghazali, Phyrrho, Mirecourt, Autrecourt, Berkeley, Hume, Montaigne, Feyerabend, Fogelikn, Van Fraassen, Gibson, Rejection of continuous, Spencer, Tyler, Teleology of life, passivity

55 *Tozzi A, Peters JF, Navarro J, Marijuán PC. 2017. Heidegger's being and quantum vacuum. Progress in biophysics and molecular biology.*https://doi.org/10.1016/j.pbiomolbio.2017.07.009.
physics, Quantum vacuum, Virtual particles, Vacuum trapping; Being, Ens, Entity, Existence, Heidegger

56 *Tozzi A, Peters JF. 2018. What it is like to be "the same"? Progress in Biophysics and Molecular Biology. 133, 30-35.* https://doi.org/10.1016/j.pbiomolbio.2017.10.005.
Descriptively near sets, BUT as universal principle, Bio-BUT; Evolution, Psychological source of identity, cell; principle of identity, Aristotle, classical logic, Sameness, Heidegger, Nagel, De Cusa, Unity, Holism, Separation

[57] *Tozzi A. 2021. Nicholas of Autrecourt: A Forerunner of Paraconsistent Logics. Preprints 2021, 2021030238. doi: 10.20944/preprints202103.0238.v1.*

58 *Tozzi A. 2019. The myth of falsifiability in the assessment of scientific theories. (electronic response to: Bellmund JLS, Gärdenfors P, Moser1EI,*

falsify your theory". This logical, Popperian tenet, dating back to the first half of the 20th Century, has been fully discarded, in particular by Lakatos, and then by Sokal, Bartley III, and so on. A scientific theory does not need to be falsifiable, rather simply requires experimentally testable, quantifiable previsions that must be treated with statistic methods to evaluate their probability. To give an example related to the scientific (not philosophical!) theory of the multidimensional brain, the "geometric codes that map information domains" can be tested by looking at the required hidden symmetries, possibly endowed in the real neurodata provided by currently-available techniques, such as EEG, fMRI.

When points and lines do not exist, de Cusa's hyperbolic geometry comes into play: a physical answer to ancient questions about infinity[59]. Starting from the tenets of human imagination, i.e., the concepts of lines, points and infinity, we provide a biological demonstration that the skeptical claim ''human beings cannot attain knowledge of the world'' holds true. We show that the Euclidean account of the point as ''that of which there is no part'' is just a conceptual device produced by our brain, untenable in our physical/biological realm: currently used terms like ''lines, surfaces and volumes'' label non-existent, arbitrary properties. We elucidate the psychological and neuroscientific features hardwired in our brain that lead us humans to think to points and lines as truly occurring in our environment. Therefore, our current scientific descriptions of objects' shapes, graphs and biological trajectories in phase spaces need to be revisited, leading to a proper portrayal of the real world's events: miniscule bounded physical surface regions stand for the basic objects in a traversal of spacetime, instead of the usual Euclidean points. Our account makes it possible to erase of a painstaking problem that causes many theories to break down and/or being incapable of describing extreme events: the unwanted occurrence of infinite values in equations. We propose a novel approach, based on point-free geometrical standpoints, that banishes infinitesimals, leads to a tenable physical/biological geometry compatible with human reasoning and provides a region-based topological account of the power laws endowed in nervous activities. Such dynamical behavior, anticipated by Nicholas de Cusa in his 1440 analogic account of "coincidentia oppositorum", can be used to describe widespread biological paths in terms of concave, flat and convex curves on a donut-like structure. We conclude that points, lines, volumes and infinity do not describe the world, rather they are fictions introduced by ancient surveyors of land surfaces.

Pairwise comparison and the infinity problem: no continuum between three apples and three bananas[60]. We provide mathematical and practice-driven justification for using [0, 1] normalization of inconsistency indicators in pairwise comparisons. The need for normalization, as well as problems with the lack of normalization, is presented. A new type of paradox of infinity is described.

When Achilles meets the Alexander horn, he leaves the straight path and overtakes the turtle[61]. Physical and biological measurements might display range values extending towards infinite. The occurrence of infinity in equations, such as the black hole singularities, is a troublesome issue that causes many theories to break down when assessing extreme events. Different methods, such as re-normalization, have been proposed to avoid detrimental infinity. Here a novel technique is proposed, based on geometrical considerations and the Alexander Horned sphere, that permits to undermine infinity in physical and biophysical equations. In this unconventional approach, a continuous monodimensional line becomes an assembly of countless bidimensional lines that superimpose in quantifiable

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Doeller CF. 2018. Navigating cognition: Spatial codes for human thinking. Science, 362(6415):eaat6766. DOI: 10.1126/science.aat6766.

Popper, falsifiability, Lakatos, Sokal, Bartley III; scientific theory; testable, quantifiable previsons

59 *Tozzi A, Peters JF. 2019. Points and lines inside our brains. Cognitive Neurodynamics, 13(5): 417–428. DOI: 10.1007/s11571-019-09539-8.*
Tozzi A. 2018. Curvature Changes in Biological Dynamics. Preprints, 2018100299 (doi: 10.20944/preprints201810.0299.v1).

Parallel transport, Straight line, Euclid geometry, Points, lines, volumes, Point-free geometry, Decision limit problem; physics, Time-reversal, Hyperbolic paths, Infinity, Observable, Continuous, Indivisibles, Quantum reality, Fullerenes; Human knowledge, Models in mind; neuroscience, brain, Demarcation, Evolutionary perception; Euclid Indivisibles, Continuous, Helmotz, Whitehead, De Cusa, Skepticism, Aristotle, Autrecourt

Closed Jordan curve, Infinity, Hopf fibration; Navier-Stokes equations; Dickinsonia, Clifford tori in CNS, Mental-framed world; Analogy, Cusa, coincidentia oppositorum, common sense

60 *Koczkodaj WW, Magnot J-P, Mazurek J, Peters JF, Rakhshani H, Soltys M, Strzałka D, Szybowski J, Tozzi A. 2017. On normalization of inconsistency indicators in pairwise comparisons. International Journal of Approximate Reasoning.86, 73–79. http://doi.org/10.1016/j.ijar.2017.04.005.*

Discrete values, Normalization, inconsistency indicators; paradox of infinity

61 *Tozzi A, Peters JF. 2020. A Topological Approach to Infinity in Physics and Biophysics. Found Sci. https://doi.org/10.1007/s10699-020-09674-0.*

Torus, Connes conjecture, infinity, von Neumann algebras, geometrical concepts of infinity, bounded as well as unbounded geometrical shapes, no point geometry, Alexander horned sphere, pathological object, wild points, Cantor set, positive-curvature manifold, negative-curvature manifold, point-free geometries, nonlinear dynamics, logistic plots ,chaotic flows, conformal infinity, generalized Heisenberg principle, quantum jump, Mandelstamm-Tamm inequality, time-energy uncertainty, singularity of black holes, extreme events, Langevin equations, Levi-Civita and Ehresmann connections, Minkowski metric, Atomism , Continuum, Achilles and the turtle, Aristotle, Al Ghazali, Averroes, Democritus, Leucippus, Epicurus, Pythagoras, Plato and Walter de Chatton Henry of Harclay, Richard Kilvington, Adam of Whodeham, Bradwardine. William of Ockham, Isaac Barrow, Leibnitz, Guillame de L'Hópital, Euler, Berkeley, Bernard Bolzano, Augustin-Louis Cauchy, Karl Weierstrass, Richard Dedekind. Georg Cantor , Brentano, Peirce, Poincaré, Brouwer, Hermann Weyl. Wittgenstein, Nicolas de Cusa

knots and bifurcations. In other words, we may state that Achilles leaves the straight line and overtakes the turtle.

ARE WE SURE THAT INFORMATION (AND INFORMATION ENTROPIES IN THE BRAIN) DOES EXIST?

Random walks are not so random, after all…[62] Physical and biological phenomena are often portrayed in terms of random walks, white noise, Markov paths, stochastic trajectories with subsequent symmetry breaks. Here we show that this approach from dynamical systems theory is not profitable when random walks occur in phase spaces of dimensions higher than two. The more the dimensions, the more the (seemingly) stochastic paths are constrained, because their trajectories cannot resume to the starting point. This means that high-dimensional tracks, ubiquitous in real world physical/biological phenomena, cannot be operationally treated in terms of closed paths, symplectic manifolds, Betti numbers, Jordan theorem, topological vortexes. This also means that memoryless events disconnected from the past such as Markov chains cannot exist in high dimensions. Once expunged the operational role of random walks in the assessment of experimental phenomena, we take aim to somewhat "redeem" stochasticity. We suggest two methodological accounts alternative to random walks that partially rescue the operational role of white noise and Markov chains. The first option is to assess multidimensional systems in lower dimensions, the second option is to establish a different role for random walks. We diffusely describe the two alternatives and provide heterogeneous examples from boosting chemistry, tunnelling nanotubes, backward entropy, chaotic attractors.

Information Entropies Are Unable to Describe Mental Activities: a mathematical alternative through the Banach–Tarski paradox[63]**.** Neuroscientists are able to detect physical changes in information entropy in the available neurodata. However, the information paradigm is inadequate to describe fully nervous dynamics and mental activities such as perception. This paper suggests explanations to neural dynamics that provide an alternative to thermodynamic and information accounts. We recall the Banach–Tarski paradox (BTP), which informally states that when pieces of a ball are moved and rotated without changing their shape, a synergy between two balls of the same volume is achieved instead of the original one. We show how and why BTP might display this physical and biological synergy meaningfully, making it possible to model nervous activities. The anatomical and functional structure of the central nervous system's nodes and edges makes it possible to perform a sequence of moves inside the connectome that doubles the amount of available cortical oscillations. In particular, a BTP-based mechanism permits scale-invariant nervous oscillations to amplify and propagate towards widely separated brain areas. Paraphrasing the BTP's definition, we could state that: when a few components of a self-similar nervous oscillation are moved and rotated throughout the cortical connectome, two self-similar oscillations are achieved instead of the original one. Furthermore, based on topological structures, we illustrate how, counterintuitively, the amplification of scale-free oscillations does not require information transfer.

What if Claude Shannon was (partially, of course) wrong[64]**?** According to Shannon, the entropy for continuous stochastic processes has many properties analogous to the entropy for discrete processes. Nevertheless, there are some mathematical and physical problems with Shannon's account. The obstacle arises in chapter 21 (Shannon 1948), when Shannon describes the continuous case of the entropy of ergodic ensemble of functions (i.e., the entropy of a set of functions together with a probability measure).

Observer's horizon & relational quantum dynamics: why entropy and information become subjective[65]**.** We describe cosmic expansion as correlated with the standpoints of local observers' co-moving horizons. In keeping with relational quantum mechanics, which claims that quantum systems are only meaningful in the context of measurements, we suggest that information gets ergodically "diluted" in our isotropic and homogeneous expanding Universe, so that an observer detects just a limited amount of the total cosmic bits. The reduced bit perception is due the decreased density of information inside the expanding cosmic volume in which the observer resides. Further, we show that the second law of thermodynamics can be correlated with cosmic expansion through a relational mechanism, because the decrease in information detected by a local observer in an expanding Universe is concomitant with an increase in perceived cosmic thermodynamic entropy, via the Bekenstein bound and the Laudauer principle. Reversing the classical scheme from thermodynamic entropy to information, we suggest that the cosmological

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62 *Tozzi A. 2021. Random Walks Are Not So Random, After All. viXra:2010.0002.*
random walks, white noise, Markov paths, stochastic trajectories, closed paths, symplectic manifolds, Betti numbers, Jordan theorem, topological vortexes, Markov chains, dynamical systems theory, boosting chemistry, tunnelling nanotubes, backward entropy, chaotic attractors.

63 *Tozzi, A., Peters, J.F. Information-devoid routes for scale-free neurodynamics. Synthese (2020). https://doi.org/10.1007/s11229-020-02895-7.*
Disjointness, ergodic, subflows, information theory, Betti numbers; against info in brain, Fourier analysis, power laws in brain, fractals, Sierpinski triangle, neuroscience, brain, physics, information entropy, Shannon, photography; duplication brain oscillation

64 *Tozzi A. 2021. Troubles with Shannon's account of the entropy in the continuous case. Vixra.*

65 *Tozzi A, Peters JF. 2019. Entropy balance in the expanding universe: a novel perspective. Entropy, 21(4), 406. https://doi.org/10.3390/e21040406.*

constant of the quantum vacuum, which is believed to provoke the current cosmic expansion, could be one of the sources of the perceived increases in thermodynamic entropy. We conclude that entropies, including the entangled entropy of the recently developed framework of quantum computational spacetime, might not describe independent properties, but rather relations among systems and observers.

CAN BE SPACE & TIME RELOADED? TOPOLOGICAL APPROACHES TO BIG BANG, BLACK HOLES AND QUANTUM DYNAMICS

A multidimensional Monster in a pre-big bang scenario: when the Multiple precedes the One in a Spinozian pre-Big Bang[66]. The Monster group, the biggest of the sporadic groups, is equipped with the highest known number of dimensions and symmetries. Taking into account variants of the Borsuk-Ulam theorem and a novel topological approach cast in a physical fashion that has the potential to be operationalized, the Universe can be conceived as a lower-dimensional manifold encompassed in the Monster group. Our Universe might arise from spontaneous dimension decrease and symmetry breaking that occur inside the very structure of the Monster Module. We elucidate how the energetic loss caused by projection from higher to lower dimensions and by the Monster group's non-abelian features is correlated with the present-day asymmetry in thermodynamic arrow. By linking the Monster Module to theoretical physical counterparts, we are allowed to calculate its enthalpy and Lie group trajectories. Our approach also reveals how a symmetry break might lead to a Universe based on multi-dimensional string theories and CFT/AdS correspondence.

Non-commutative unification of relativity and quantum dynamics: Connes comes into play[67]. The unexploited unification of general relativity and quantum physics is a painstaking issue that prevents physicists to properly understanding the whole of Nature. Here we propose a pure mathematical approach that introduces the problem in terms of group theory. Indeed, we build a cyclic groupoid (a nonempty set with a binary operation defined on it) that encompasses both the theories as subsets, making it possible to join together two of their most dissimilar experimental results, i.e., the commutativity detectable in our macroscopic relativistic world and the noncommutativity detectable in the quantum, microscopic world. This approach, combined with the Connes fusion operator, leads to a mathematical framework useful in the investigation of relativity/quantum mechanics relationships.

Quantum entanglement can be solved in four spatial dimensions[68]. A quantum entanglement's composite system does not display separable states and a single constituent cannot be fully described without considering the other states. We introduce quantum entanglement on a hypersphere - which is a 4D space undetectable by observers living in a 3D world -, derived from signals originating on the surface of an ordinary 3D sphere. From the far-flung branch of algebraic topology, the Borsuk-Ulam theorem states that, when a pair of opposite (antipodal) points on a hypersphereare projected onto the surface of 3D sphere, the projections have matching description. In touch with this theorem, we show that a separable state can be achieved for each of the entangled particles, just by embedding them in a higher dimensional space. We view quantum entanglement as the simultaneous activation of signals in a 3D space mapped into a hypersphere. By showing that the particles are entangled at the 3D level and un-entangled at the 4D hypersphere level, we achieved a composite system in which each local constituent is equipped with a pure state. We anticipate this new view of quantum entanglement leading to what are known as qubit information systems.

Black holes horizons: an unnoticed correlation with the Borsuk-Ulam theorem and the Mobius strip[69]. The Möbius strip spacetime topology and the entangled antipodal points on black hole surfaces, recently described by 't Hooft, display an unnoticed relationship with the Borsuk-Ulam theorem from algebraic topology. Considering this observation and other recent claims which suggest that quantum entanglement takes place on

66 *Tozzi A, Peters JF. 2016. Symmetries, Information and Monster Groups before and after the Big Bang. Information 7(4), 73; doi:10.3390/info7040073.*
J-function, Leech lattice, Sporadic groups, Fisher-Griess Monster Group, Energy-BUT, Re-BUT, general-BUT; physics, Monstrous moonshine, Symmetries, Lie group, Pre-big bang, symmetry break, string theories, CFT/AdS correspondence; Spinoza, Badiou,

67 *Tozzi, A.; Peters, J.F.. Unification of Commutative Relativity and Noncommutative Quantum Dynamics Via Cyclic Groupoids and Spacetime Fusion Categories. Preprints 2018, 2018090021 (doi: 10.20944/preprints201809.0021.v1).*
Dagger category, subset, Cyclic groupoids, Commutativity, Non-commutativity; Connes fusion categories and operator; physics, spacetime fusion categories, deformed special relativity, Deformation quantization techniques, Unification quantum dynamics and relativity; Holism, Unification

68 *Peters JF, Tozzi A. 2016. Quantum Entanglement on a Hypersphere. Int J Theoret Phys, 1–8. doi:10.1007/s10773-016-2998-7.*
Hypersphere, dimensions; BUT; physics, Quantum information theorem, Qbit information system, Entanglement, separable states; Multidimensional world

69 *Tozzi A, Peters JF. 2019. Topology of Black Holes' Horizons. Emerging Science Journal, 3(2):58-63. DOI: 10.28991/esj-2019-01169.*
Mobius strip, Olographic-BUT, Mobius-BUT, general-BUT, BUT as universal principle; physics, Entangled horizon, Olographic principle, Black holes horizon, Negative mass, Entanglement timeless in 3D; superobservers; t'Hooft

the antipodal points of a S3 hypersphere, a novel topological framework can be developed: a feature encompassed in an S2 unentangled state gives rise, when projected one dimension higher, to two entangled particles. This allows us to achieve a mathematical description of the holographic principle occurring in S2. Furthermore, our observations let us to hypothesize that a) quantum entanglement might occur in a four-dimensional spacetime, while disentanglement might be achieved on a motionless, three-dimensional manifold; b) a negative mass might exist on the surface of a black hole.

The geometry of black holes in terms of random walks[70]. The Universe, rather than being homogeneous, displays an almost infinite topological genus, because it is punctured with a countless number of gravitational vortexes, i.e., black holes. Starting from this view, we aim to show that the occurrence of black holes is constrained by geometric random walks taking place during cosmic inflationary expansion. At first, we introduce a visual model, based on the Pascal's triangle and linear and nonlinear arithmetic octahedrons, which describes three-dimensional cosmic random walks. In case of nonlinear 3D paths, trajectories in an expanding Universe can be depicted as the operation of filling the numbers of the octahedrons in the form of "islands of numbers": this leads to separate cosmic structures (standing for matter/energy), spaced out by empty areas (constituted by black holes and dark matter). These procedures allow us to describe the topology of an universe of infinite genus, to assess black hole formation in terms of infinite Betti numbers, to highlight how non-linear random walks might provoke gravitational effects also in absence of mass/energy, and to propose a novel interpretation of Beckenstein-Hawking entropy: it is proportional to the surface, rather than the volume, of a black hole, because the latter does not contain information.

General relativity vs. quantum mechanics: a matter of manifold[71]? The unexploited unification of general relativity and quantum mechanics (QM) prevents the proper understanding of the micro- and macroscopic world. Here we put forward a mathematical approach that introduces the problem in terms of negative curvature manifolds. We suggest that the oscillatory dynamics described by wave functions might take place on hyperbolic continuous manifolds, standing for the counterpart of QM's Hilbert spaces. We describe how the tenets of QM, such as the observable A, the autostates ψa and the Schrodinger equation for the temporal evolution of states, might work very well on a Poincaré disk equipped with rotational groups. This curvature-based approach to QM, combined with the noncommutativity formulated in the language of gyrovectors, leads to a mathematical framework that might be useful in the investigation of relativity/QM relationships. Furthermore, we introduce a topological theorem, termed the punctured balloon theorem (PBT), which states that an orientable genus-1 surface cannot encompass disjoint points. PBT suggests that hyperbolic QM manifolds must be of genus ≥ 1 before measuring and genus zero after measuring. We discuss the implications of PBT in gauge theories and in the physics of the black holes.

An atomic geometric model encompassing quantum mechanics' dictates too[72]. Here we provide a novel atomic, paraxial model in which a single belt of electrons surrounds the nucleus. The electronic belt is depicted in terms of broken lines and split wavy trajectories that intersect an axis, giving rise to small angles that can be accurately calculated. We demonstrate that the probabilistic electronic cloud of the atom described by quantum mechanics can be depicted in terms of an electronic belt, because its sizes closely match the descriptions given by de Broglie and Heisenberg. In touch with the claims of the two latter Authors, the wavy trajectories around the nucleus come back to a starting point, so that their orbits are stationary.

WHAT IS A BORDER? CAN THE PHYSICAL REALITY BE DEMARCATED?

The problem of boundaries and borders: from Medieval thinkers to the contemporary physics and biology[73]. When a boat disappears over the horizon, does

70 *Tozzi A, Yurkin A, Peters JF. 2019. Cosmic Random Walks Underlying an Infinite-Genus Universe. Preprints, 2019030237.*
Betti numbers, Infinite topological genus, Pascal triangle, octahedron, island of numbers, Vortex cycles; physics, Cosmic expansion, inflation, hyperuniformity, Lloyd iterations, Holographic principle critique, Beckenstein-Hawking entropy, Black hole horizon; black hole does not encompass information, rather it is empty

71 *Tozzi A. 2021. Formulation Of Quantum Mechanics On Poincare Disks. Preprints 2021, 2021030533 (doi: 10.20944/preprints202103.0533.v1.*
Tozzi A. 2021. Lack of Disjointness in Genus-1 Surfaces: The Punctured Balloon Theorem. viXra:2101.0005.
punctured balloon theorem, general relativity, quantum mechanics postulates, hyperbolic continuous manifolds, Hilbert spaces, autostates, Poincaré disk, black holes

72 *Yurkin A, Peters JF, Tozzi A. 2018. A novel belt model of the atom, compatible with quantum dynamics. Journal of Scientific and Engineering Research, 2018, 5(7):413-419. ISSN: 2394-2630.*
Paraxial models; physics, Atom, Electronic belt, Quantum dynamics, probability; De Broglie, Heisenberg

73 Tozzi A. 2020. Are Borders Inside or Outside? Found Sci. https://doi.org/10.1007/s10699-020-09708-7.
Borsuk-Ulam theorem, Borders, Internal and external, decision limit problems, bounded objects with fuzzy edges, infinity; boundary; Jordan curve theorem, four-dimensional torus, stereographic projection of a Clifford torus, Lévy's zero–one law, projectionism, closeness of sets of points with their

a distant observer detect the last moment in which the boat is visible, or the first moment in which the boat is not visible? This apparently ludicrous way of reasoning, heritage of long-lasting medieval debates on decision limit problems, paves the way to sophisticated contemporary debates concerning the methodological core of mathematics, physics and biology. These ancient, logically-framed conundrums throw us into the realm of bounded objects with fuzzy edges, where our mind fails to provide responses to plain questions such as: given a closed curve with a boundary (say, a cellular membrane) how do you recognize what is internal and what is external? We show how the choice of an alternative instead of another is not arbitrary, rather points towards entirely different ontological, philosophical and physical commitments. This paves the way to novel interpretations and operational approaches to challenging issues such as black hole singularities, continuous time in quantum dynamics, chaotic nonlinear paths, logarithmic plots, demarcation of living beings. In the sceptical reign where judgements seem to be suspended forever, the contemporary scientist stands for a sort of God equipped with infinite power who is utterly free to dictate the rules of the experimental settings.

Is the world inside or outside your mind? Is your mind inside or outside the world?[74]. We describe cosmic expansion as correlated with the standpoints of local observers' co-moving horizons. In keeping with relational quantum mechanics, which claims that quantum systems are only meaningful in the context of measurements, we suggest that information gets ergodically "diluted" in our isotropic and homogeneous expanding Universe, so that an observer detects just a limited amount of the total cosmic bits. The reduced bit perception is due the decreased density of information inside the expanding cosmic volume in which the observer resides. Further, we show that the second law of thermodynamics can be correlated with cosmic expansion through a relational mechanism, because the decrease in information detected by a local observer in an expanding Universe is concomitant with an increase in perceived cosmic thermodynamic entropy, via the Bekenstein bound and the Laudauer principle. Reversing the classical scheme from thermodynamic entropy to information, we suggest that the cosmological constant of the quantum vacuum, which is believed to provoke the current cosmic expansion, could be one of the sources of the perceived increases in thermodynamic entropy. We conclude that entropies, including the entangled entropy of the recently developed framework of quantum computational spacetime, might not describe independent properties, but rather relations among systems and observers.

When holes define objects:
a) How to solve decision limit problems... with fuzzy holes[75]. Starting from unidentified objects moving inside

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neighbourhoods. Jordan Curve Theorem, No points algebra, descriptive nearness, point-free geometries, topological vortices, Betti numbers, physics, continuous time in quantum dynamics, chaotic nonlinear paths, logarithmic plots, infinite-dimensional Hilbert space, material plenum, separate, continuously extended vacuum, event horizon of a black hole, singularity inside the black hole, wormholes, null singularity at the Cauchy horizon, anti-reference, Einstein's relativity, Relational quantum mechanics, superposition and entanglement, Collective systems, non-equilibrium dynamics, self-organized criticality that operates at the edge of chaos, metastable state of second-phase transitions, spontaneous avalanches, universal power laws, Chaotic systems display dependence from initial conditions, positive Lyapunov exponents, attractors, metastable state of second-order phase transitions, infinite correlation length, countless functional dimensions, spontaneous avalanches and universal power laws, logistic map, bifurcation diagram, exponential and the linear plot, active capacity (force), passive capacity (resistance), natural and artificial vacuum, pressure, heat, black hole singularities, metrology, dynamic systems theory, cellular automata, stochastic activation rate, mean first passage time, Brownian particle, gradient/descent apparatuses of Bayesian machines, mean time of the first among many arrivals" of particles at the target, Statistics of the extreme, optimal random search, unconventional superconductivity, quantum computing, demarcation of living beings, cellular membrane, Cytoplasm, central and peripheral nervous system, proteins and nucleic acids, liquid–liquid phase separation , compartmentalization of cellular components. living beings, spermatozoon to reach the egg, gene topology, HIV proteins arrangement, Sceptical, God with infinite power, Bradwardine, Nicholas of Autrecourt, Aristotelian analytic arguments, Offredus. "incipit" and "desinit", the "beginning" and the "end", Medieval scholars, Oxonians, Oxford, University of Paris, primo et ultimo istanti (the first and the last instant), and incipit and desinit (the beginning and the end), Heytesbury, maximum and minimum magnitudes, Autrecourt, maximum quod non, Beginning, **borders,** Blasius of Parma, Heytesbury, Albert of Saxony, Buridan, celestial spheres, Oresme, Liber sex principiorum, Porretano, Primum Mobile, Augustine, the Being and the Good, the Evil and the Not-Being, principle of non-contradiction hold, Buridan, Potentia Dei infinita, Potentia Dei Ordinata, Isaac Newton, hypotheses non fingo

74 *Tozzi A, Peters JF. 2019. Entropy balance in the expanding universe: a novel perspective. Entropy, 21(4), 406. https://doi.org/10.3390/e21040406.*
physics, Relational quantum mechanics; cosmic expansion, local observer commoving horizon, ergodicity, isotropic, homogeneous; bit; information; second law of thermodynamics; thermodynamic entropy, information, Bekenstein bound, the Laudauer principle; Rovelli

75 *Tozzi A, Peters JF. 2020 Removing uncertainty in neural networks. Cognitive Neurodynamics. https://doi.org/10.1007/s11571-020-09574-w.*
Tozzi A. 2019. Does horizontal transmission amend our concept of living species? (electronic response to: Chen J, Quiles-Puchalt N, Chiang YN, Bacigalupe R, Fillol-Salom A, et al. 2018. Genome hypermobility by lateral transduction. Science, Vol. 362, Issue 6411, pp. 207-212. DOI: 10.1126/science.aat5867).
Disconnectedness, Persistent homology, Boundary, Jordan curve, Jordan theorem, Set theory, decision limit problem, topological changes; Cosmic bodies, neural networks; Species definition, Denisovans, Neandertals, Fire propagation, migration flows; Set theory, Disconnected objects, Uncertainty, Autrecourt
Decision limit problems, boundaries, Jordan curve theorem; Markov blanket; Horizontal transmission, Species definition, Species-specific barriers, Evolution, Homo Sapiens, Neandertals, Denisovians, Floresians

a two-dimensional Euclidean manifold, we propose a method to detect the topological changes that occur during their reciprocal interactions and shape morphing. This method, which allows the detection of topological holes development and disappearance, makes it possible to solve the uncertainty due to disconnectedness, lack of information and absence of objects' sharp boundaries, i.e., the three troubling issues which prevent scientists to select the required proper sets/subsets during their experimental assessment of natural and artificial dynamical phenomena, such as fire propagation, wireless sensor networks, migration flows, neural networks' and cosmic bodies' analysis.

b) Holes: what for in the brain? The occurrence of Betti numbers in neurodata[76]. spatio-temporal brain activities with variable delay detectable in resting-state functional magnetic resonance imaging (rs-fMRI) give rise to highly reproducible structures, termed cortical lag threads, that can propagate from one brain region to another. Using a computational topology of data approach, we found that Betti numbers that are cycle counts and the areas of vortex cycles covering brain activation regions in triangulated rs-fMRI video frames make it possible to track persistent, recurring blood oxygen level dependent (BOLD) signals. Our findings have been codified and visualized in what are known as persistent barcodes. Importantly, a topology of data offers a practical approach in coping with and sidestepping massive noise in neuro data, such as unwanted dark (low intensity) regions in the neighbourhood of non-zero BOLD signals. A natural outcome of a topology of data approach is the tracking of persistent, non-trivial BOLD signals that appear intermittently in a sequence of rs-fMRI video frames. The end result of this tracking of changing lag structures is a persistent barcode, which is a pictograph that offers a convenient visual means of exhibiting, comparing and classifying brain activation patterns.

c) Holes: what for in the brain? The inhibitory cavities of the brain in the geometric terms of Pascal's triangles[77]. The brain, rather than being homogeneous, displays an almost infinite topological genus, since it is punctured with a high number of "cavities". We might think to the brain as a sponge equipped with countless, uniformly placed, holes. Here we show how these holes, termed topological vortexes, stand for nesting, non-concentric brain signal cycles resulting from the activity of inhibitory neurons. Such inhibitory spike activity is inversely correlated with its counterpart, i.e., the excitatory spike activity propagating throughout the whole brain tissue. We illustrate how Pascal's triangles and linear and nonlinear arithmetic octahedrons are capable of describing the three-dimensional random walks generated by the inhbitory/excitatory activity of the nervous tissue. In case of nonlinear 3D paths, the trajectories of excitatory spiking oscillations can be depicted as the operation of filling the numbers of octahedrons in the form of "islands of numbers": this leads to excitatory neuronal assemblies, spaced out by empty areas of inhibitory neuronal assemblies. These mathematical procedures allow us to describe the topology of a brain of infinite genus, to represent inhibitory neurons in terms of Betti numbers and to highlight how spike diffusion in neural tissues is generated by the random activation of tiny groups of excitatory neurons. Our approach suggests the existence of a strong mathematical background subtending the intricate oscillatory activity of the central nervous system.

Betti numbers, Infinite topological genus, Pascal's triangle, octahedron, island of numbers, Vortex cycles; random walks; hyperuniformity, Lloyd iterations, Holographic principle critique, excitatory/inhibitory ratio, E/I ratio

[76] *Don AP, Peters JF, Ramanna S, Tozzi A. 2020. Topological View of Flows inside the BOLD Spontaneous Activity of the Human Brain. Front. Comput. Neurosci. DOI: 10.3389/fncom.2020.00034*
Topological inference from spontaneous activity structures in FMRI videos with persistence barcodes.
Betti numbers, Mitra fMRI, video frame

[77] *(Tozzi A, Yurkin A, Peters JF. 2021. A Geometric Milieu Inside the Brain. Found Sci. https://doi.org/10.1007/s10699-021-09798-x.)*