
# TOWARDS TOPOLOGICAL MECHANISMS UNDERLYING EXPERIENCE ACQUISITION AND TRANSMISSION IN THE HUMAN BRAIN

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**ABSTRACT**

Experience is a process of awareness and mastery of facts or events, gained through actual observation or second-hand knowledge. Recent findings reinforce the idea that a naturalized epistemological approach is needed to further advance our understanding of the nervous mechanisms underlying experience. This essay is an effort to build a coherent topological-based framework able to elucidate particular aspects of experience, *e.g.*, how it is acquired by a single individual, transmitted to others and collectively stored in form of general ideas. Taking into account concepts from neuroscience, algebraic topology and Richard Avenarius' philosophical analytical approach, we provide a scheme which is cast in an empirically testable fashion. In particular, we emphasize the foremost role of variants of the Borsuk-Ulam theorem, which tells us that, when a pair of opposite (antipodal) points on a sphere are mapped onto a single point in Euclidean space, the projection provides a description of both antipodal points. These antipodes stand for nervous functions and activities of the brain correlated with the mechanisms of acquisition and transmission of experience.

Current advances in human neurosciences shed new light on questions concerning the status of mental activity and its relation to physical function. Promising innovations such as transcranial stimulation (Filmer et al., 2014) and mind-to mind communication (Grau et al., 2014) call for broad methodological investigations in order to further advance our understanding of the brain. In this essay we will focus on *experience*, an active process of understanding, knowledge and mastery of facts or events which encompasses a different range of brain functions, such as thought, perception, memory, emotion, will, imagination, inter-individual communication and scientific theories (Cavell, 2002). Experience is gained through: a) the direct, actual observation of immediately perceived events and subsequent interpretation, b) or records or summaries from first-hand observers or experiencers c) or from instruments (Popper and Eccles, 1997).

In such a framework, simple concepts from algebraic topology, e.g., the Borsuk-Ulam theorem (BUT) and its variants, come into play, giving rise to a different approach that makes it possible to evaluate mental activities correlated with experience. In this essay, we will limit our topological evaluation to specific features of experience, *e.g.*, its acquisition and transmission. We will start from the pure, unmediated experience of the single individual and will proceed towards objects categorization and social experience through a plurality of shared experiences. In other words, we will investigate knowledge acquisition by an individual, a community and mankind at large. We will pursue the inductive approach introduced by the French Swiss philosopher Richard Avenarius (1843-1896). His *Kritik der reinen Erfahrung* (*Critique of Pure Experience*) (Avenarius, 1908) provides a useful high-level interpretation of physiological functions and psychological behavior, which works well in building a topological framework for experience (Russo Krauss, 2013; Russo Krauss, 2015).

This paper comprises six sections. The first section introduces BUT and its variants. Section two, devoted to objective evidence from experiments, shows how, despite our current lack of topological knowledge of the brain activity, a large number of papers tackled the issues of geometric representations of perceptual phenomena, relationships between geometrical shapes and nervous function, and changes in brain dimensions. The third section analyses the experiences of a single individual, who includes no other standpoint than that where he stands. As the case of the Greek philosopher, he is in the turmoil of the market-place, not as buyer or seller, in order that he may just observe. Particular cases are provided in sections four and five, in order to establish the psychological correlates of experience acquisition and transmission. In section six, we extend the experience onto social groups and a general human setting, where the most general concepts may arise. Section seven aims to answer the crucial question: what does a topological approach bring to the table, in the experimental evaluation of physiological mechanisms of experience?

## 1) THE BORSUK-ULAM THEOREM AND ITS VARIANTS

The Borsuk-Ulam Theorem (BUT) states that, if a sphere $S^n$ is mapped continuously into an *n*-dimensional Euclidean space $R^n$, there is at least one pair of antipodal points on $S^n$ which map onto the same point of $R^n$ (Borsuk, 1933; Dodson, 1997; Beyer, 2004) (**Figure 1A**). See Tozzi and Peters (2016a) for further details and a mathematical treatment. The notation $S^n$ stands for an *n*-sphere (Weeks, 2002), which is an *n*-dimensional, circular structure, embedded in an *n+1* space (Marsaglia, 1972). For example, a 2-sphere ($S^2$) is the 2-dimensional surface of a 3-dimensional space (the exterior of a beach ball is a good illustration). Antipodal points are, *e.g.*, the poles of a sphere (Matousek, 2003; Collins, 2004).

We can consider regions on an *n*-sphere that are either adjacent or far apart (**Figure 1B**). This means that pairs of points need not be antipodal, in order to have matching descriptions (Peters, 2016). This BUT variant has utility, provided there are a pair of regions on an *n*-sphere which are *similar*. The concept of antipodal points can be used not just for the description of points, but also for more complicated structures, such as shapes of space (object contours), temporal intervals (temporal oscillations), functions, vectors and symmetries (Borsuk, 1958-59; Borsuk, 1969; Peters, 2014; Tozzi and Peters 2016b) (**Figure 1C**). This leads to a particularly useful region-based BUT, dubbed reBUT (Peters, 2016). We are thus allowed to describe brain features on an *n*-sphere either as antipodal points, or antipodal regions. This means that antipodal signal shapes can be compared (Weeks, 2002; Peters, 2016). That is, feature-based descriptions of antipodes can be assessed at one level of observation, while their projections into a single shape can be analyzed at a lower level (Tozzi and Peters, 2016b).

Although BUT was originally limited to an *n*-sphere where *n* is a natural number, nevertheless *n* can also be regarded as rational or irrational number (Tozzi and Peters, 2016b). For example, we might regard functions or shapes as embedded in a sphere in which *n* stands for number of instants in time. Hence, the parameter *n* becomes useful in the description of dynamical systems such as the brain.

The original formulation of BUT describes the presence of antipodal points on spatial manifolds in every dimension, provided the manifold is a convex, positive-curvature structure (e.g., a ball). However, the kind of geometry that best describes human sensation, perception and higher mental processes, such as knowledge and consciousness, is a matter of debate. Theoretical contenders able to connect the stimulus and mental issues include Euclidean, elliptic and hyperbolic geometry. This means that brain functions might occur on manifolds endowed with other types of geometry: for example, the hyperbolic one encloses complete Riemannian *n*-manifolds of constant sectional curvature -1 and concave shape (*i.e*, a saddle) (Sengupta et al., 2016). We are thus allowed to look for antipodal points also on structures equipped with kinds of curvature other than the convex one (Mitroi-Symeonidis, 2015). Or, in other words, whether the brain displays a concave, convex or flat functional appearance, it does not matter: we may always find the points with matching description predicted by BUT (Tozzi, 2016). For further details, see also Peters and Tozzi (2016b).

In sum, the BUT displays four versatile ingredients which can be modified in different guises: a continuous mapping, two antipodal (or non-antipodal) points (or functions) with matching description, an *n*-sphere where the *n* value may change and, last but not least, the mapping of antipodal points into the lower level of a *n-1* sphere.

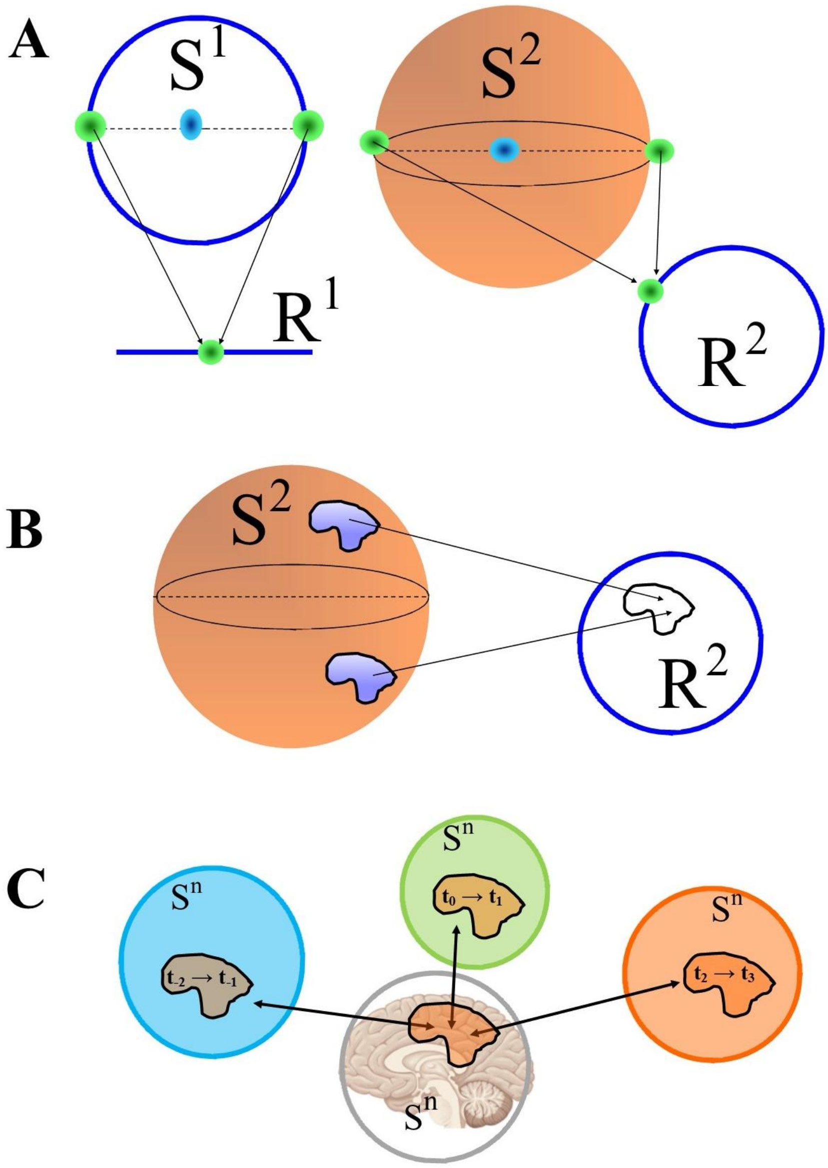


***Figure 1A.*** *The Borsuk-Ulam theorem in case of $S^n$ being a circle and a sphere. Two antipodal points in $S^n$ project to a single point in $R^n$, and vice versa.* ***Figure 1B.*** *Simplified sketch of a BUT variant. Two functions with matching description do not need either to be antipodal (shown here), or embedded in the same n-sphere (shown in* **Figure 1C***), in order to be described together.* ***Figure 1C*** *displays an example of brain oscillations in a topological framework. In this case, the antipodal regions are three temporal intervals, embedded in different spatial n-spheres, which project to the brain surface. Therefore, the concept of similar antipodal signals can be used not just for the description of points or spatial features, but also of temporal sequences. This means that BUT allows us to compare not just motionless points or shapes, but also events.*

## 2) NEUROSCIENCE AND TOPOLOGY: STATE OF THE ART

The use of topological and geometrical tools (such as projections, shapes and vectors) for the assessment of mental phenomena dates back to the pioneering works of Alfred North Whitehead (Whitehead 1919) and Kurt Lewin (Lewin, 1935 and Lewin, 1936). In more recent times, such approach has been put forwards by "neuro-episthemological" scholars and pursued, among others, by eliminative materialism (Churchland, 2007), integration information theory (Tononi, 2008), while Suppes (2002) provides a setting for investigating problems of representation and invariance in any systematic part of science. Due to the current lack of knowledge and the novelty of this field, it must be clearly stated that the "smoking gun" describing whether the invariant isomorphic structure between the brain and its function is effective or not, has not yet been found.

However, some studies assessed with proper experimental procedures the transformational relationship between the stimuli and the mechanisms of mental phenomena such as experience, knowledge, sensation, cognition, consciousness. They point towards a topological approach to mental phenomena as objective evidence, obtained either from actual experiments or results of computer simulations. Luneburg's task (1947) was to assess the mapping of physical space into visual space. This task can be fulfilled, if the metric of visual space can be expressed in guise of a physical bipolar coordinate system. The highly influential studies from Marr (1982) addressed the issue of visual information in terms of mathematical analysis and computational investigation. Findings suggest that nervous structures process information through topological as well as spatial mechanisms. For example, experiments on tachistoscopic perception of visual stimuli demonstrated that the visual system is sensitive to global topological properties, indicating their extraction as a basic factor in perceptual organization (Zeeman 1962). It is has been hypothesized that hippocampal place cells create topological templates to represent spatial information (Dabaghian et al., 2012; Dabaghian et al., 2014; Arai et al., 2014; Chen, 2014). Using a computational model based on Persistent Homology Theory, Milton et al. (2015) demonstrated that, in the physiological range, hippocampal place cells encode the topological features of the environment that serves as a basis of a cognitive map of space, spatial memory and spatial awareness. We will show in the next sections how the novel incarnations of the "classical" Borsuk-Ulam theorem lead to a better comprehension and assessment of several topological features of direct perception and imagination.

It has been argued that perception of persisting surfaces depends on the perception of specific structures, invariant over time (Gibson, 1971; Gibson, 1979). Gibson accounted for our knowledge of world objects by borrowing a concept of invariance in topology: a series of transformations can be endlessly and gradually applied to a pattern without affecting its invariant properties (Gibson,1950). Recognition (perceiving) objects entails continuous mappings between our representations of objects and in-the-world ones (Johnson-Laird, 2010). In effect, perception can be explained by equivalent configurations (form, shapes) of objects (Heft 1997; Rock, 1983). And, principal among the properties of world objects, is shape and acquisition of persistent perception of object shapes, which we explain with BUT variants.

The raising interest for topology in order to investigate neural structures is testified by a series of recent published papers. For example, Giusti et al. (2015) developed a novel approach to matrix analysis, called clique topology, in order to extract both random and geometric meaningful structures in pyramidal neurons in rat hippocampus, during non-spatial behaviors such as wheel running and rapid eye movement sleep. The Authors suggest that the geometric structure of correlations is shaped by the underlying hippocampal circuits, and does not stand merely for a consequence of position coding. Simas et al. (2015) devised a method that allows different operations between networks which share the same set of nodes, by embedding them in a common metric space and enforcing transitivity to the graph topology. They applied this method to construct an aggregated network from a set of functional graphs, each one from a different subject. Therefore, the comparison of aggregated functional topological network reveals emerging features, that could not be observed when the comparison is performed with the classical averaged functional network. Robinson et al. (2016) found that the eigenmodes of a single brain hemisphere's spatial structure are close analogues of spherical harmonics, which are the natural modes of the sphere. This last finding allows the use of the classical BUT in the description of neural field theory.

A foremost argument related to brain topology is the symmetry. Symmetries are invariances underlining physical and biological systems (Weyl, 1982). A symmetry break occurs when the symmetry is present at one level of observation, but "hidden" at another level (Roldàn et al., 2014). BUT tells us that we can find, on an *n*-dimensional sphere, a pair of opposite points that have the same encoding on an *n*-1 sphere. This means that symmetries can be found when evaluating the system in a proper dimension, while they disappear (are hidden or broken) when we assess the same system in just one dimension lower (Tozzi and Peters, 2016b). The term dimension, in our case, does not stand for a spatial dimension, rather reflects functional relationships of brain activities that take into account the dimensionality of the neural space. Connectivity and complex network analyses of neural signals allow the assessment of brain activity's dynamics, providing a novel insight into the multidimensionality of various neural functions' representations (Luce et al., 1995; Kida et al., 2016; Robinson et al. 2016). Apart from giving insights in neural dynamics in the *canonical* three dimensions (space, time, and frequency), complex network analyses are also able to evaluate other functional dimensions, e.g. categories of neuronal indices such activity magnitude, connectivity, network properties and so on (Kida et al., 2016). From a dynamical system perspective, one would expect that nervous dynamics can be described in terms of trajectories and/or manifolds in n-dimensional phase spaces (Lech et al., 2016). Mazzucato et al (2016) demonstrated that stimuli reduce the dimensionality of cortical activity. Clustered networks, such as default mode network, have instead a larger dimensionality, because the latter grows with ensemble size: the more neurons are recruited, the more the dimensions (Mazzucato et al., 2016). Singer et al. (2016) proposed that the low-dimensional cortical dynamics due to stimulus-induced synchronization in the visual cortex are sub-states that represent the results of computations performed in the high-dimensional state-space provided by recurrently coupled networks. Because dimension reduction and symmetry breaking display close relationships, symmetries are correlated with changes in

functional dimensions in the brain. Indeed, a key feature of dynamical approaches is that the dynamics they predict are characterized by non-equilibrium phase transitions, and therefore breaks of symmetries (Scholz et al., 1987; Jirsa et al., 1998). Brain symmetric states display dimensions higher than the asymmetric ones. In such a vein, Stemmler et al. (2015) proposed that animals can navigate by reading out a simple population vector of grid cell activity across multiple, different microscopic dimensions. While the spatial activity of a single grid cell does not constitute a metric, an ensemble of hierarchically organized grid cells does provide instead a distance measure (Stemmler et al., 2015).
In sum, the study of changes in brain dimensions proposed by BUT and its variant is a promising novel methodology. For example, Benson et al. (2016) recently developed an algorithmic framework for studying how complex networks are organized by higher-order connectivity patterns, revealing unexpected hubs and geographical elements. Furthermore, Kleinberg et al. (2016) demonstrated that real networks are not just random combinations of single networks, but are organized in specific ways dictated by hidden geometric correlation between layers. Such correlations allowed the detection of multidimensional communities, e.g., sets of nodes that are simultaneously similar in multiple layers. Crucial for our topological arguments, such multidimensionality also enables accurate trans-layer link assessment, meaning that connections in one layer can be predicted by observing the hidden geometric space of another layer. For example, when the geometric correlations are sufficiently strong, a multidimensional framework outperforms navigation in the single layers, allowing efficient targeted navigation simply by using local multilayer knowledge (Kleineberg et al., 2016).

## 3) EXPERIENCE ACQUISITION AT THE INDIVIDUAL LEVEL

Every human individual originally accepts:

a) an ever-changing spatial environment composed of manifold parts dependent on one another,
b) other human individuals making manifold describable statements and
c) these statements are dependent upon the environment.

When an individual becomes aware of some of the manifold parts of the environment outside him, he states he is having an experience. In a topological framework, we embed the environment in an *n*-sphere, the cortical layers in a *n*-Euclidean space and the describable statements in another *n*-sphere (**Figure 2**). The environmental *n*-sphere contains an object detected by the human individual, *e.g.*, manifold parts perceived together, while the other *n*-sphere contains statements which are not simple sounds or noises, but words (or gestures, such as the lacrimal secretions which point to crying or mimic movements) describing the object. A triad is accomplished, where a person is the only one of the three member who is a witness. The object and its verbal counterpart stand for two antipodal points, while their matching description is achieved in the brain of the human individual. It is noteworthy that the topological relationships among the three members do not exist out of the triad (**Figure 3**). We cannot perceive the thoughts and the sensations of other people, but just assign them thoughts and sensations analogous to ours. In topological words, we project our thoughts, emotions and statements to other people embedded in further environmental n-spheres. Our perception of an object continues, even when the object is out of sight. This can concisely explained by viewing regions on the surface of a $S^n$ sphere as multiple representations of object shapes, mapped continuously to an object embedded in the environment that we have seen and continue to see. This occurs thanks to the continuous mapping from shapes in our memory to shapes in Euclidean space. In effect, persistence perception can be viewed as signals matching, i.e., real-scene visual signals that are collectively the umbra of physical shapes. Therefore, a mutual relationship occurs between the individual and the environment (Gibson, 1986). This perceptive experience is direct and immediate: information is directly taken from the environment, with no mediations. The topological concept of a smart mechanism able to extract the invariants from the input stream means that the active subject puts in relationships and connexions the objects through his senses, with no need for mental processing in order to build them. Topology nicely explains this interesting feature, which is common not just to perceptions, but also to higher mental activities (see the next paragraphs). Indeed, the antipodal points do not stand just for spatial features, such as an object, but also for temporal sequences (**Figure 1C**). Therefore, the brain links together not just spatial elements in an object, but also temporal events in a concept, an idea, a proposition, a hope, a desire and so on. Furthermore, the individual can be embedded not just in a spatial *n*-sphere, but also in an *n*-sphere in which *n* stands for the time, according to the dictates of the BUT variants. Many spheres with different times are possible: this means that the individual is able to link past and future events.

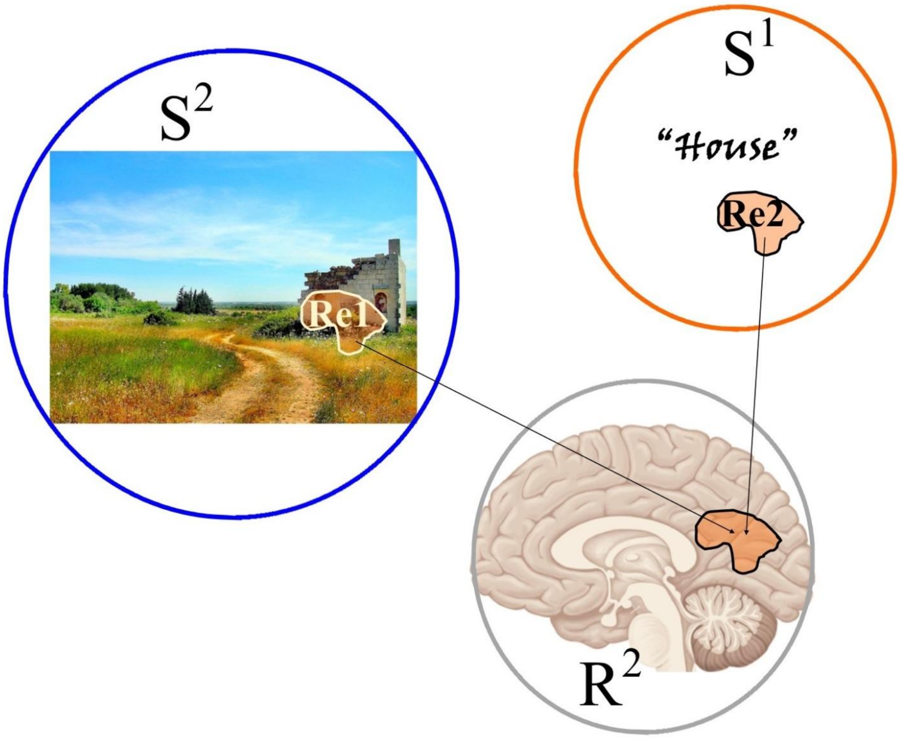


***Figure 2***. Topological view of the individual experience evaluated through BUT variants. The environment is embedded in a $S^n$ sphere, where the signal Re1, standing for an object, displays matching description with the signal Re2 embedded in a $S^{n-1}$ sphere, standing for the word corresponding to the object. Both Re1 and Re2 project to a single signal, embedded in a manifold $R^n$, e.g., the brain.

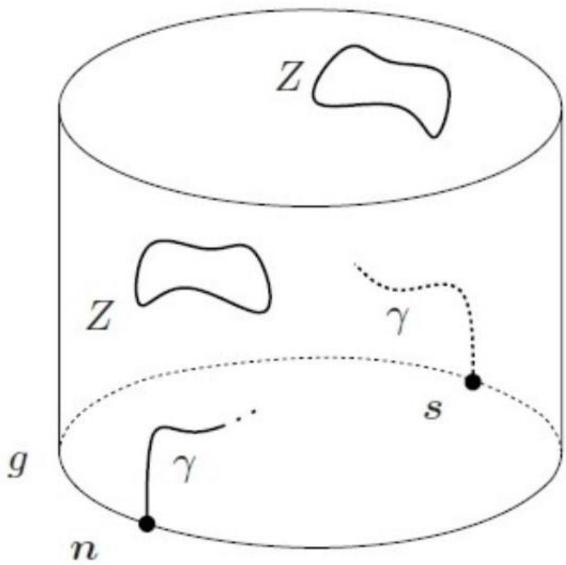


***Figure 3***. *The two figures Z with matching description project to the two functions γ on the g plane. It easy to see that, if we follow the two functions γ from n and s on, their ends cannot meet. Likewise, the two figures Z display similar features, but are disjoint. Modified from Matousek, 2003.*

Consider the following propositions, that are neither equivalent to each other nor are they interchangeable:

a) if a statement comes from the environment, then the statement is an experience;
b) if a statement is an experience, then the statement comes from the environment.

Indeed, the link between environment and statements is very dynamical. Sometimes, when environment changes, *e.g.*, during a chemo-physical experiment, a variation in statement follows. Other times statements may vary even if environment does not change, and vice versa: *e.g*., over time, a child watching the same figure could detect modifications in the image, or the the same name may refer to different people. Statements may also change due to differences in the cultural background of individuals.

From this, we can ask the following three questions.

1) In what sense does an experience come from the environment?
2) In what sense is a statement an experience?
3) Do these two questions collimate or diverge?

The next Sections will be devoted to the answers of these three questions, via topological tools. The concepts of matter, substance, soul, consciousness, necessity, freedom, reality will not be included in our answers, nor the long-standing issue of causality. Indeed, the topological concept of proximity helps in elucidating the cause-and-effect problem in observing phenomena. Current scientific experience is grounded on the tenets of causality, *e.g.*, the relation between two processes where the first, the cause, is assumed to be responsible for the second, the effect. It is believed that the brain infers the causes underlying sensory events (Parise et al., 2011), or, in other words, is able to understand the causes when just the effects are known. This inferencing is not limited to conscious and high-level cognition, but it is also performed continually and effortlessly in unconscious perception (Körding et al. 2007). We argue that, if we evaluate different spatial and temporal (neural) phenomena in the light of the "sameness" and projections instead of causality, we achieve a novel view of brain events which has the potential to be operationalized and cast in an empirical way. Inference is replaced by affine connections, proximities, mappings and homotopies (i.e., perceived shapes are mapped into others, such as a coffee cup mapped to a torus). This is in touch with some philosophers (Hume, 1739-40) and scientists (Libet et al., 1987; Bengson et al., 2014) who believe that inference is just a trick of the mind, *i.e.*, the cause-effect relationships do not exist, but are merely apparent correlations among events dictated by our natural, evolutionary instinct of mental association among conjoined events. In Hume's words, we tend to believe that things behave in a regular manner to the extent that behavior patterns of objects seem to persist into the future, and throughout the unobserved present. Chicharro and Ledberg (2012) noticed that, because subsystems in the brain are often bidirectionally connected (Friston, 2008; Barrett and Simmons, 2015), this means that interactions rarely should be quantified in terms of cause-and-effect. A recently introduced concept, termed primitive chaos, is closely related to fundamental problems such as determinism, causality, free will and predictability (Ogasawara, 2010; Ogasawara and Oishi, 2012). Under some conditions, a new primitive chaos is constructed from the original primitive one, achieving a hierarchic structure. Such a picture implies that new events and causality are built from the old ones through a coarse-graining process. It is noteworthy that, under natural conditions, each primitive chaos leads to topological transitivity, while their infinite varieties are guaranteed by a non-degenerate Peano continuum, which typically might stand for a $S^n$ sphere (Ogasawara and Oishi, 2014; Ogasawara, 2015).

## 4) EXPERIENCE MECHANISMS: BRAIN OSCILLATIONS TOWARDS UNSTABLE ENERGETIC EQUILIBRIA

A *system* is formed of two variables connected in a way such that when one changes, the other changes. The initial state occurs before the variation, the final state afterwards. **Figure 4** provides an example of how a topological formulation of a system allows an objective quantification of empirically testable features. The changes can be either real or merely possible. The final state depends on two settings, namely, the initial state of the system and the external factors acting on the system.

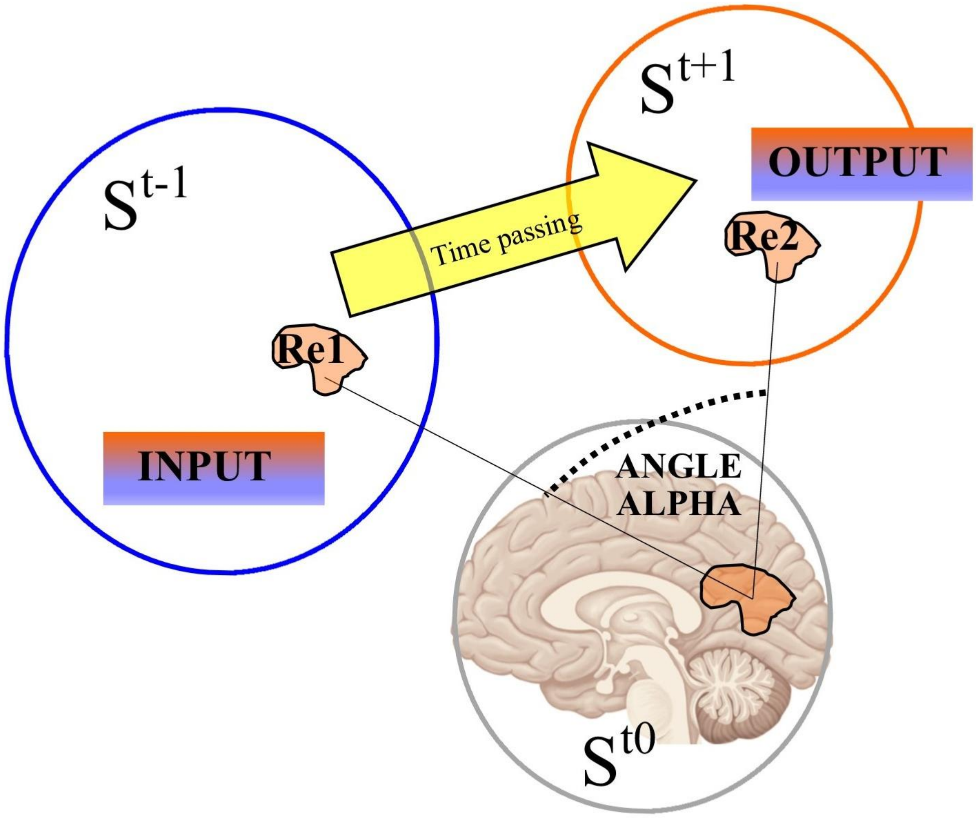


***Figure 4****. A system formed by n- spheres made of time dimensions, according to one of the BUT variant. The antipodal regions Re1 and Re2 are located respectively onto a sphere equipped with a dimension of time T-1 (the past) and T+1 (the future). The brain occupies the present time T0. The angle alpha, corresponding to the temporal distance between the presentation of the stimulus and its word retrieval, can be measured. Modified from Tozzi and Peters (2016b).*

The environment of a single individual is also a system that can be divided into spatial and social environments. The environment acts on organisms in two ways, namely, impinging on an organism causing a material change, or stimulating the nerves. A human individual is the source of a peculiar environment, having both non-nervous and nervous systems. The nervous system, in turn, can be divided in peripheral and central nervous system (Nieuwenhuys et al, 2008). The statements made by human beings do not depend on the environment, nor by the peripheral nerves. Indeed, when nerves are damaged, it is still possible to have sensations such as phantom limb syndrome and visual hallucinations during optic nerve atrophy. Is there a part of the CNS which provides a basis for statements? We need not care about the precise anatomical and physiological structure, since experimental evidence has not sufficiently localized brain functions. Therefore, it makes sense to agree that there is a division of functional subsystems in the brain, irrespective of differences in morphological development and functional relationships. From this point forward, the term *brain* refers to unidentified subsystems that are the source of statements.

Acceptance that statements depend on the environment is tantamount to accepting that they depend on the brain and, particularly, on changes in brain. Statements depend on the environment just in the sense that the environment is able to modify the brain. While variations of the environment are studied by the special sciences, we will focus instead on variations of an individual, and in particular, of his brain. Statements do not directly depend on the environment but rather on brain variations. An entire psychical life is a function of self-preservation of the organism within certain limits. The highest levels of conservation are achieved when an energetic equilibrium occurs in the brain between two opposite dynamical forces, namely, brain activity and energy requirements. The brain alternates between these states of equilibrium and breaks into a process that consists of *oscillations* between the two phases, in the deviation from a preliminary value and in approximating to it again. When changes take place, the equilibrium of a brain is disturbed and conditions that annul the changes occur, so that the brain approximates once again to its maximum-maintenance. The functional phenomenon termed brain *oscillation* might encompass different mechanisms, *e.g.*, attractors (Tozzi, Fla, Peters 2016), transient heteroclinic channels (Afraimovich et al., 2013), networks with collective computational abilities

(Hopfield 1982; Izhikevich, 2010), nervous oscillators (Zlotnik et al., 2016) and so on. Despite the countless possible scenarios, the processes governing the brain *oscillations* may be generalized, when we take into account the second law of thermodynamics, which states that *every process occurring in nature proceeds in the sense in which the sum of the entropies of all bodies taking part in the process is increased* (Planck's formulation). An issue of central relevance in this context is the *free-energy principle* (FEP) (Friston, 2010). The brain is a self-organizing system at non-equilibrium steady state with its environment (Tognoli and Kelso, 2014) which has to minimize its free-energy, in order to resist a natural tendency towards disorderly state and high entropy. This formulation reduces the physiology of brain to its homeostasis, namely, the maintenance of system states confronted with a constantly changing environment. The brain is regarded as an active inference machine that works according to Bayesian principles: sensory inputs constrain estimates of prior probability from past experience, to create the posterior probabilities that serve as beliefs about the causes of such inputs in the present (Sengupta et al., 2013; Sengupta et al, 2014: Sengupta et al., 2016). The probability of sensory states must have low entropy and, since entropy is also the average self-information or "surprise", the brain implicitly avoids surprises (Friston et al, 2015). Agents suppress free-energy (or surprise) by changing: a) the sensory input, though an action on external states (the environment), or b) their internal brain states, through perception or attention. The imperative to maintain a non-equilibrium steady-state through exchanges with the environment acquires a logical-mathematical framework when evaluated through topological lenses. Indeed, one can regard belief-updating as free-energy minimisation in terms of energetic gradients and information flows (Sengupta et al., 2016) occurring in brain phase spaces embedded in n-spheres. We will be back on the operationalization of this energetic argument in Section five. The concepts of *proximity* (spatial as well as descriptive closeness) and affine connections are also helpful in solving one of the problems raised by *oscillations, e.g.,* how cooperation among so many "distant" sub-networks occurs seamlessly in real time. Thanks to the continuous mapping provided by BUT, we achieve a quantifiable explanation to the speed and balance of a system characterized by hierarchical and cross-hierarchical cooperating modules (Linkenkaer-Hansen et al., 2001).

## 5) VITAL TRAINS, CHARACTERS, ELEMENTS

In the previous Section, we argued that the elementary physiological processes consist of a state of disturbance of the equilibrium of the brain, followed by a restoration of the difference. All the changes which lie between this beginning and end of a physiological process are termed *vital train.* This is a complex three-stages process. The initial section emerges from a quiet state in which the oscillation, *e.g.*, a disturbance which breaks the equilibrium, is introduced. Follows then an intermediate section, where compensation mechanisms are activated via variations counteracting the oscillations. The final state is such that the oscillation is suppressed, with return to the quiet and maximum preservation state. The removal of the alteration, in order to achieve the highest level of conservation, may occur in two ways, namely, return to the original stable state or to a different stable state. Here we give an example which illustrates how mental life is arranged in three sections. At start, there is the expected value, stated by the individual as sure, true, known. When a statement is perceived as the same, long chains of oscillations lead to *all together*, *always*, *everywhere*, progressively reaching more general concepts. When the *identical* becomes a rule, the rules become laws and the truths become sure, existing, while the *different* becomes exception. Then enters a variation, stated as different, diverging, doubtful, unexpected. It comes together with the feeling of pain, opposition, or uncertainty, dissatisfaction. The second, following section is an effort to suppress the anomaly: the brain strives to remove the unpleasant experience. In touch with FEP, variations can be removed either through peripheral action (practical behavior), or through brain action (theoretical behavior). There are two ways to suppress the variations: (1) reduction of the unknown to the known; (2) gradual habituation to a change, so that the unknown becomes known. In the fifth Section we will describe how topology helps in elucidating these ways to suppress variations. A value is achieved in the final section which leads to feelings of rest, satisfaction, certainty, truth and quiet. It must be stressed that each statement is influenced by type and magnitude of an individual's background (Betzel et al., 2014). Although every vital train of thoughts ends with phase three, the ideal equilibrium of the brain can be never achieved, due to the continuous dynamics of infinite, superimposed, cross-linked, hierarchical vital trains. Indeed, the brain operates at the edge of chaos and tends to live near a metastable state of second-phase transition (Papo, 2014; Deco and Jirsa, 2012; Beggs and Timme, 2012; de Arcangelis and Herrmann, 2010). In other words, the brain displays nonlinear dynamics. As we will see in Section five, BUT and its variants give us the possibility to achieve linear dynamics from nonlinear ones.

A vital train is a process that progressively removes useless terms, approximating to the final pure stages equipped with: a) minimum energy and maximum stability, b) higher spatial and temporal unrestriction. The evolution of vital trains is towards more perfect series, *i.e*., the lowest energetic basins, where the gradient descent values can be calculated by comparing angles between n-spheres.

The role of habituation and its changes is crucial, because they give rise to the *identity* and the *difference*. The more the same oscillation is repeated, the more the sensation is perceived as usual, and the same, certain, the known, the familiar. In an energetic context, habituation stands for the state equipped with the lowest possible energy (Tozzi et al., 2016). The final state of the brain is a physiological configuration which is unvaried towards every changing component of the environment. However, this final state embeds not just the final pure state, but also other accidental factors, which need to be progressively abolished. This occurs because minimal energy is difficult to reach. The answer to the first question in Section 2 is thus the following: the environment is at the base of the experience just in case it leads to variations in the individual brain, and just in case a statement depends on environmental variations. However, such values are valid just for a given individual, in a given moment: the individual final state varies from individual to individual, and also from place to place and from epoch to epoch (Barttfeld et al., 2015; Gorgolewski et al., 2014; Vuksanovic and Hövel, 2014).

In sum, the general scheme is the following: when an individual makes a statement, a change in oscillations occurs in brain. The statements depend upon the characteristics of the physiological variations in brain and its oscillations. Of utmost importance is the sensation of prevalence and perception of finer details (discrimination), in which some sensations are highlighted by the attention, compared with the faded background sensations. In topological terms, the prevalence plays an important role in deciding which parts of the environmental n-sphere must be taken into account: it means that the brain chooses from the external inputs the complex of elements of biological significance and marks them as *signals*. A possible role for visual attention and prevalence in an energetic gradient descent context has been recently hypothesized (Sengupta et al., 2016). Consciousness can be linked to our first BUT ingredient, namely, a continuous mapping. Indeed, brain activity needs continuous inputs from a source of enduringly active neurons. The ascending arousal system is a good candidate, because it contains tonic active neurons which guarantee a continuous electric stream towards the cortex, at least during alertness (Nieuwenhuys et al., 2008). In such a framework, consciousness stands both for physiological tonic neurons and for the continuous mapping required by BUT.

Human statements can be partitioned into characters (qualifications like pleasant, nice, beneficial, unpleasant) and elements (such as green, cold, hard, sweet). The difference between characters and elements is that the former are mutable and can be defined as forms, while the latter are permanent and can be defined as contents. At first, we examine experience regarded as character, *i.e.* the variable form (emotions, sensations, perceptions and so on), in which we experience anything. The question is not what individuals mean by an experience, but rather what individuals state is an experience. Many examples can be listed in which statements are referred as experience, *e.g*., the appearance of the sun, perceived temporal relationships among events, the difference between sleep and arousal, travel in far countries during dreams, miracles, presages of death, causal connections, footprints of animals, calculations and mathematical theorems. So, what is an experience? Is it always linked to the external environment? Does it refer to entities, or simply to the cognition of their existence, or is it pure cognition without existence? No one of the solutions proposed in answer to these questions is satisfactory, because experience cannot be defined in only in one way. The individual simply bumps into the components of the environment, and then describes what he has encountered. When a little girl states that she has seen angels, then the angels are an experience for her. The individual who states "it is an experience", is equivalent to coalescing a plurality of single perceptions, explainable thanks to BUT and its variants.
Thus, a better although incomplete definition of experience could be: *something perceived*. Also the ego, the I, is a statement, in which perception is felt as the individual's own perception. The more an observable and the individual are separated, the more the observable is perceived as active and a factum of experience, while the individual perceives selfhood as a passive percipient. Now we evaluate experience regarded as content, *i.e*., everything which is felt, sensed, perceived and so on. When individuals state they have an experience, what is the content of their experience? That content depends on modifications in the brain and on the current state and education of each individual. Regardless whether statements are about existing or non-existing entities, all the values perceived in a given moment as experience are indeed the content of experience, and may vary with individual changes. The answer to the second question raised in Section 2 is thus the following: experience, in a narrower definition, is the perception from the brain's standpoint of the things extracted from the environment. A wider definition of experience does not involve things, but ideas. Ideas can also appear to be about existing, simply there or given entities, so that, even if a centaur is not an experience, its idea is an experience. Thus, a *continuous* flow occurs from the perceived thing (the narrower concept of experience) to the represented idea (the wider concept of experience). *Continuos*, in touch with one of the BUT ingredients.

### 6) EXPERIENCE TRANSMISSION AT INTER-INDIVIDUAL LEVELS

Brains of different individuals join together in higher order systems formed by social assemblies. The whole mass of human experience tends to become adapted as much as possible to the surrounding world. Humanity stands, as a whole, for a kind of ultra-human organism, following the same basic rules of self-preservation of the individual. When the geographical and social environment widens to include the entire Earth surface and all mankind, the brain removes the accidental factors via a process of progressive elimination. However, due to the differences of initial conditions among individuals, the ideas achieved in the third section could lead, after a while, once again from a known to an unknown. The ultimate suppression of variations occurs via the search for universal laws. Over time, individuals and the generations project faster, simpler and unavoidable concepts. This ongoing conceptualization of our experience is an approximation process that always tends towards identical highly generalized, unlimited concepts characterized by simple, complete descriptions, by qualitative differences expressed in many ways and by formal equivalences expressed in different forms.

The highest concepts, i.e. the final states corresponding to all the possible environment manifold parts, are equipped with the maximum possible frequency and embrace all the possible historical contents of the statements. For a topological explanation, see **Figure 5A**. Godel's suggestion of abstract terms more and more converging to the infinity in the sphere of our understanding take us straight to a comparison with systems equipped with BUT antipodal points and regions. The highest concept, expressed by the statement the *whole is this* is the *concept of the world* (e.g., an abstract idea). In terms of BUT, we achieve a progressive symmetry decrease. Starting from the countless symmetries endowed in the environment, we accomplish into the brain their substantial decrease (Tozzi and Peters, 2016b).

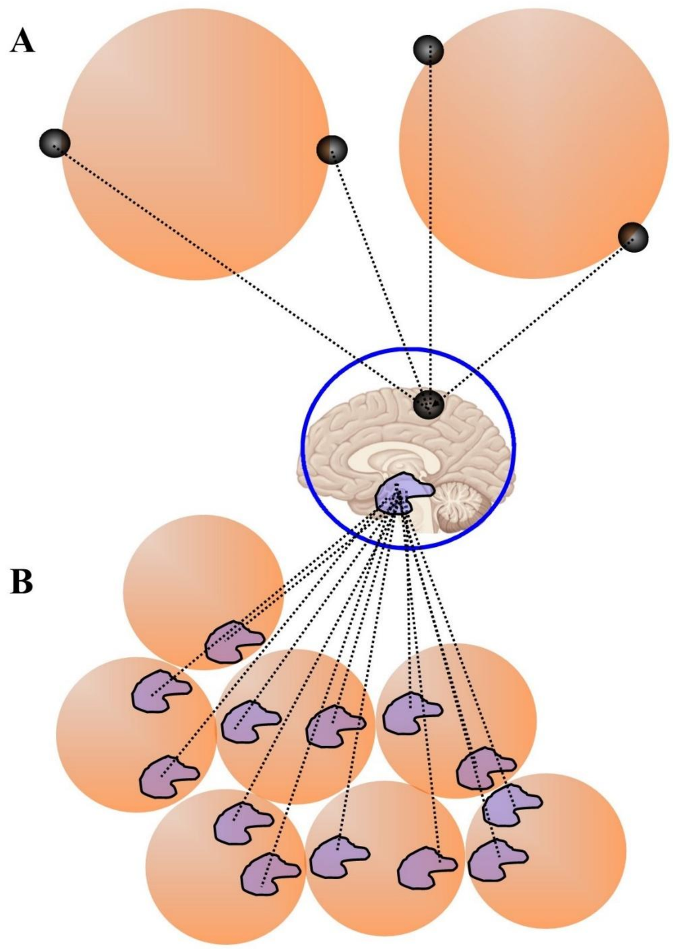


***Figure 5****. Convergence of n-spheres' antipodal points with matching description.* ***Figure A****: the ideas (abstractions) link together antipodal points embedded in different environmental n-spheres. Their projections to the brain give rise to abstract concepts.* ***Figure B****: the final, pure abstractions encompass countless different matching functions from different, closely packed, environmental n-spheres.*

However, the concept of the world still embraces more general concepts as well as superfluous ones. Indeed, many different conceptualizations of the world result from genetic constraints, social groups and historical background. The concepts of the world are still incomplete, because they also include unnecessary components and their historical development has not ended at this time.

The concept of the world is preserved by individuals and left to posterity via communication and conditioning. If a concept is transmitted unchanged over generations, its strength decreases with the time due to habituation, until it cannot be recognized anymore. The transformation of the concepts of the world leads from certainty to doubts about them. Thus arises the *enigma* of the world, in place of the concept of the world. Confronted with the enigmas of our environment, the more perplexity increases, the more urgent is the need to solve the problems as soon as possible and several new different satisfying concepts of the world may arise. Solutions are however transitory and changing with the time, both within individuals and communities. Every concept of the world is characterized by residual unneeded concepts, depending on the education of the individuals. The number of superfluous concepts may either remain the same, or decrease, or increase. Provisional solutions to the enigma of the world are countless and the only way to find more general concepts is to expunge the surplus components. The more the concept of the world is spatially enlarged, the more the solution is general and definitive, approximating an exclusive experience. Increasingly over space and time, concepts of the world tend towards the higher level concepts and the brain is satisfied. During cultural evolution, the concepts of the world are at first regarded as a positive experience, a perceived thing, an empiric experience. However, in a following phase of removal, they weaken and are regarded as non-empirical experience, pseudo-problems, until they gradually vanish and disappear. The last phase is their reappearance in a distilled form, just as pure descriptions (**Figure 5B**). More mature, pure concepts of the world allow the brain to be entirely conditioned by the environment and mature statements are regarded as complete experience. There is an answer to the third question raised in Section 2. That is, a primitive experience (experience=naturally perceived) leads to addition of non-experience (either experience=perceived and experience=not perceived) until a definitive, just described, exclusive experience takes place (experience=purified perceived). More advanced social groups, *e.g.*, scientists, may be closer than others to pure concepts of the world, at least in some fields of experience. Indeed, evolution does not proceed linearly nor uniformly. Small groups of individuals have an important role in pushing forward changes that lead to possible solutions of the problems.

## 7) WHY BRING TOPOLOGY INTO THE ASSESSMENT OF EXPERIENCE?

The question here is: what does a topologic reformulation add in the evaluation of the nervous processes of experience? BUT and its extensions provide a methodological approach which makes it possible for us to study experience in terms of projections from real to abstract phase spaces. The importance of projections between environmental spaces, where objects lie, and brain phase spaces, where mental operations take place, is also suggested by a recent paper, which provides a rigorous way of measuring distance on concave neural manifolds (Sengupta et al., 2016). The real, measurable nervous activity can be described in terms of paths occurring on n-spheres. It leads to a consideration of affinities among nervous signals, characterized as antipodal points on multi-dimensional spheres embedded in abstract spaces. To provide an example, **Figure 4** shows how embedding brain activities in n-spheres allows the quantification of geometric parameters, such as angles, lengths, and so on, that could be useful in neuroimaging data optimization. BUT and its ingredients can be modified in different guises, in order to assess a wide range of nervous functions. Although this field is nearly novel and still in progress, with several unpublished findings, we may provide some examples. Such a methodological approach has been proved useful in the evaluation of brain symmetries, which allow us to perform coarse- or fine-grained evaluation of fMRI images and to assess the relationships, affinities, shape-deformations and closeness among BOLD activated areas (Tozzi and Peters, 2016b). Further, BUT has been proved useful in the evaluation of cortical histological images previoulsy treated with Voronoi tessellation (Peters et al., 2016).

A wide range of brain dynamics, ranging from neuronal membrane activity to spikes, from seizures to spreading depression (Wei; Bernard; Richardson), lie along a continuum of the repertoire of the neuronal nonlinear activities which may be of substantial importance in enabling our understanding of central nervous system function and in the control of pathological neurological states. Nonlinear dynamics are frequently studied through logistic maps equipped with Hopf bifurcations, where intervals are dictated by Feigenbaum constants. Tozzi and Peters (2016b) introduced an approach that offers an explanation of nervous nonlinearity and Hopf bifurcations in terms of algebraic topology. Hopf bifurcation transformations (the antipodal points) can be described as paths or trajectories on abstract spheres embedded in n-spheres where n stands for the Feigenbaum constant's irrational number (Kim; Schleicher). Although the paper takes into account just Hopf bifurcations among the brain nonlinear dynamics, this is however a starting point towards the "linearization" of other nonlinear dynamics in the brain. In sum, BUT makes it possible for us to evaluate nonlinear brain dynamics, which occur during knowledge acquisition and processing, through much simpler linear techniques.

BUT and its variants are not just a *methodological* approach, but also display a *physical*, quantifiable counterpart. To make an example, although anatomical and functional relationships among cortical structures are fruitfully studied, *e.g.*, in terms of dynamic causal modelling, pairwise entropies and temporal-matching oscillations (Friston, 2010; Watanabe et al., 2014), nevertheless *proximity* among brain signals adds information that has the potential to be operationalized. For example, based on the ubiquitous presence of antipodal cortical zones with co-occuring BOLD activation, it has been recently suggested that spontaneous brain activity might display donut-like trajectories (Tozzi and Peters 2016a).
BUT allows the evaluation of energetic nervous requirements too. There exists a physical link between the two spheres $S^n$ and $S^{n-1}$ and their energetic features. When two antipodal functions a n-sphere $S^n$, standing for symmetries, project to a *n*-Euclidean manifold (where $S^{n-1}$ lies), a single function is achieved and a symmetry break occurs (Tozzi and Peters 2016b). It is known that a decrease in symmetry goes together with a decrease in entropy. It means that the single mapping function on $S^{n-1}$ displays energy parameters lower than the sum of two corresponding antipodal functions on $S^n$. Therefore, in the system $S^n$ and $S^{n-1}$, a decrease in dimensions gives rise to a decrease in energy. We achieve a system in which the energetic changes do not depend anymore on thermodynamic parameters, but rather on affine connections, homotopies and continuous functions. A preliminary example is provided by a recent paper, where BUT allows the detection of Bayesian Kullback-Leibler divergence during unsure perception (Tozzi and Peters, 2016b).

## CONCLUSIONS

In conclusion, we have provided a topological approach to experience features which makes it possible to evaluate the interactions among environment, brain and human statements. A shift in conceptualizations is evident in the BUT approach. The interaction of neural signals is described in terms of *sameness*, *closeness* and their *feature value vectors* (*e.g.*, amplitude, duration, and so on). The invaluable opportunity to treat elusive mental activities in terms of topological structures makes it possible for us to describe experience in the language of powerful analytical tools. Embracing the approach provided by BUT and its variants in characterizing brain interactions means that the *real* neuronal activity can be described as paths or trajectories on *abstract* structures. This takes us into the realm of computational topology, a realm endowed with abstract metric spaces, *e.g.* projections of environmental spaces into geometric ones. It supplies us with richly sufficient statistics, that helps us to elucidate the mechanisms of human experience.
We conclude with a methodological remark. The symbolic logic approach to scientific experience, prevailing in the beginning of the twentieth century after Whitehead and Russell's influential book (1910) and the rise of logical positivism, was dismissed in the following decades. Our novel approach is paradoxically a return to the past, because it leads once again to emphasize the old weapons of logic thinking as methodologies able to inspire scientists in their experimental evaluation of physical/biological counterparts of human experience.

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