

TOWARDS PLASMA-LIKE COLLISIONLESS TRAJECTORIES IN THE BRAIN

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Plasma studies depict collisionless, collective movements of charged particles. In touch with these concepts, originally developed by the far-flung branch of high energy physics, here we evaluate the role of collective behaviors and long-range functional couplings of charged particles in brain dynamics. We build a novel, empirically testable, brain model which takes into account collisionless movements of charged particles in a system, the brain, equipped with oscillations. The model is cast in a mathematical fashion with the potential of being operationalized, because it can be assessed in terms of McKean-Vlasov equations, derived from the classical Vlasov equations for plasma. A plasma-like brain also elucidates cortical phase transitions in the context of a brain at the edge of chaos, describing the required order parameters. In sum, showing how the brain might exhibit plasma-like features, we go through the concept of holistic behavior of nervous functions.

In the true plasma, i.e., the fourth state of matter displayed at high energies, the particles' collective behavior is governed by: a) the reciprocal influences among the nearby charged particles embedded in a so called "Debye sphere", and b) by a plasma parameter, e.g., the average number of particles in the sphere (Sturrock, 1994; Goldston and Rutherford, 1995). In

plasma, the spontaneous formation of spatial features takes place on a wide range of length scales on the boundary of a metastable state, giving rise to collisionless particles' collective movements (Cheng and Gamba 2012). A slight change in just one or two parameters is able to give rise to completely different particle trajectories and collisionless patterns.

We would not call brain function with the term *plasma*, but we will use instead the term *plasma-like*. The reason of our terminological choice is that, in true plasma, long-range correlations are preserved in order to avoid the large, disruptive electromagnetic fields which develop in presence of net charge. Furthermore, large systems of interacting particles should exhibit typical Debye lengths in order to be classified as “plasma” systems, and such parameter is still unknown in the brain. In its place, in brain we are in front of nearly neutral charges, e.g., a quasi-neutral state with slow to moderate electromagnetic fields and forces. Nevertheless, a similitude between brain and plasma collective dynamics is feasible, because, on temporal scales slower than plasma frequencies (about 10^{11} Hz) and at room temperature, plasma may be thought as a neutral fluid in which spontaneous electric fields are null. This description fits with the features of the human brain, that is equipped with a ratio between excitatory and inhibitory neurons collectively equaling the zero (Haider et al., 2006; Lombardi et al, 2012; Xue et al, 2014) and with long-range correlations. Experimental and theoretical clues allow us to conjecture that collisionless particles movements occur in the brain and that some nervous activities could be mediated by long-range extracellular flows (Linkenkaer-Hansen et al., 2001; Vuksanovic and Hövel, 2014; Gollo et al., 2017). It has been proposed that fluxes occurring in the complex intracellular web of molecular interactions could represent a universal feature of metabolic activity in living cells (Rossi et al., 2011; Prasad et al., 2014; Almaas et al, 2016). By implementing a flux balance analysis of an *Escherichia coli* strain, Almaas et al. (2016) showed that, whereas most metabolic reactions have low fluxes, the overall metabolic activity is dominated by reactions characterized by high ones. Cells respond to changes in growth conditions by reorganizing the rates of selected fluxes, predominantly within such a high-flux backbone. Li et al. (2015) suggested the presence of specific band-limited mechanisms of interregional oxygen correlation, driven by rhythmic or pseudo-oscillatory mechanisms. Traveling waves in humans may underlie processes involved in cortical activity’s global coordination. For example, Alexander et al. (2016) evaluated whole head MEG, recorded during an observer-triggered apparent motion task. Episodes of globally coherent activity occurred in the delta, theta, alpha and beta bands of the signal in the form of large-scale waves, which propagated with a variety of velocities. Their mean speed at each frequency band was proportional to temporal frequency, giving a range of 0.06 to 4.0 m/s, from delta to beta. The wave peaks moved over the entire measurement array, during both ongoing activity and task-relevant intervals. This means that a large proportion of the cortical signal measurable at the scalp occurs as large-scale coherent motion. (Alexander et al., 2016).

Equations describing collisionless movements have been already used in the evaluation of large neuronal networks. Taking into account nets’ specificity, topology, spatial extension and resulting space-dependent delays, Touboul

(2012) developed a neural model based on stochastic analysis methods. He analyzed the collective behavior of mean-field limits of large-scale, spatially extended stochastic neuronal networks with delays. The asymptotic regime of such systems is characterized by integral-differential McKean–Vlasov equations. The latter are similar to usual neural field equations, but, because they incorporate noise levels as a parameter, they allow analysis of noise-induced transitions (Touboul, 2012). Deriving tractable reduced equations of biological neural networks, in order of capturing the macroscopic dynamics of neuronal sub-populations, has been a long-standing problem in computational neuroscience. Delarue et al. (2015) proposed a reduction of large-scale multi-population stochastic networks, based on the mean-field theory. They investigated the well-posedness of a networked integrate-and fire model describing an infinite population of neurons which interact with one another through their common statistical distribution. Inglis and Talay (2015) evaluated the convergence of a stochastic particle system that interacts through threshold hitting times towards a novel equation of McKean–Vlasov type. In this novel neurons’ network model, a classical noisy integrate-and-fire model is coupled with a cable equation, in order to describe the dendritic structure of each neuron (Inglis and Talay, 2015). Furthermore, Galtier and Touboul (2013) derived, for a wide class of spiking neuron models, a system of differential equations of the type of the usual Wilson-Cowan systems. It describes the macroscopic activity of populations, under the assumption that synaptic integration is linear with random coefficients. Simulations of the reduced model showed a clear-cut agreement with real networks’ macroscopic dynamics (Galtier and Touboul, 2013). In our paper, we will make a slightly different use of the McKean–Vlasov type equations, because they will be encompassed in a more general context: a brain function independent from subtending networks or large neural assemblies.

MATERIALS AND METHODS

The rationale. We hypothesize that the electromagnetic fields and currents generated by cortical neuronal spikes and other sources could be equipped with Vlasov-like, collisionless interactions. Such long-range interactions stand for an electrically quasi-neutral brain medium of unbound positive and negative particles. Classically, the relationship between an extracellular potential V_e (measured in Volts) and the current source density (measured in $A\ m^{-2}$) at a particular point of brain tissue is given by the Maxwell and Laplace equations (Buszaki et al., 2012). It is believed that multiple currents and sources combine linearly by the superimposition principle. However, if we take into account a plasma-like brain, the classical Ohmic equations are valid just in the narrow spatial limits of the Debye length, while, at a distance $\gg \gg \gg$ Debye sphere (we might speculate that a value of $\gg \gg \gg \gg$ Debye length needs to be at least in the range of centimeters in the brain), the time evolution of

the charged particles' distribution function is not anymore Maxwellian. Long-range interactions take place, which can be mathematically evaluated in terms of collisionless equations. Indeed, a self-consistent collective electromagnetic field is produced by charged ions, so that the time evolution of long-range interactions among particles could be described by the Vlasov-Maxwell system of equations. In order to evaluate brain functions in a plasma-like context, we need to remove, from the classical Vlasov equation for highly charged plasma (Vlasov 1938), the useless (in our context) term which describes the distribution function for electrons, and to leave just the one for ions. In such a way, we are allowed to describe the dynamics of the brain as a system of charged particles in an electromagnetic field. With a suitable statistical treatment, the behavior of a large ensemble of random walks can be described by the equations of macroscopic diffusion (deBuyl et al., 2011). Therefore, a modified diffusion equation could describe the collective trajectories of the electromagnetic spikes in the brain (Syková and Nicholson, 2008).

Building a plasma-like model for brain activity. Here we introduce modified Vlasov equations lacking its *classical* term of free electrons: the so-called McKean-Vlasov equations. Indeed, applications of McKean-Vlasov model goes far beyond plasma physics and astrophysics, being now widely used also in biosciences (Bellomo et al., 2017). The latter equations, instead of mean charge moments, display number density, flow velocity and pressure moments (Dawson and Vaillancourt, 1995; Chayes and Panferov, 2010; Kotelenz and Kurtz, 2010). Note that these three variables can be described in terms of means of main observables and are functions just of position and time. In such a way, a self-consistent collective field comes into play, representing brain long-range behavior. Therefore, the brain currents display collective, plasma-like long range trajectories.

It has been recently proposed that mental operations, such as spontaneous activity and mind wandering, follow constrained, topological donut-like trajectories along preferential functional brain railways, shaped as a torus (Tozzi and Peters, 2016). In order to describe the movements of such trajectories, the plasma-like brain model is very helpful. If we depict brain fluxes as taking place on a torus, we notice that the collisionless trajectories can be described as movements in the toroidal space. If we now take into account equations on a n -torus, we can describe the time-evolution of probabilistic distribution of densities and their free energy functional (Chayes and Panferov, 2010). This treatment in terms of collisionless toroidal movements can be assessed through the McKean-Vlasov equation, that describes the time-evolution of a density $\rho = \rho(x, t)$, where $x \in \mathbb{T}_L^d$, the d -dimensional torus of scale L . The equations states that:

$$\rho_t = \Delta_x \rho + \theta L^d \nabla_x \cdot \rho \Delta_x (V * \rho),$$

where V is a real-valued function of x , with the meaning of interaction potential. In addition, ρ is the density, θ the

interaction (coupling) strength, e.g., the underlying density of the fluid, L is the torus scale, d the torus dimension, and $*$ denotes the convolution in x . L^d stands for the volume factor. The term ∇ denotes the divergence or the curl (rotation) of a vector field defined on a multi-dimensional domain. We now need to evaluate the brain counterparts. The value $\rho(x, t)$ has a probabilistic interpretation which we attribute to particle or the well-known value of fluid brain density. The value V stands for the interaction potential. It is assumed that ρ integrates to one. The dynamics is a gradient flow (with respect to a certain distance in the space of probability measures) for the *free energy* functional F . This means that, in touch with a long tradition of studies quantifying brain energy (Friston 2010; Sengupta et al., 2013), our model is also able to assess the required dynamics and free energy functional. Indeed, the dynamical equation can be recast into the form:

$$\frac{d}{dt} F_\theta(p) = - \int_{\mathbb{T}_L^d} \rho \left| \nabla \log \frac{\rho}{e^{-\theta L^d V * \rho}} \right|^2 dx,$$

where the dynamics of the McKean-Vlasov equation is a gradient flow for $F_\theta(p)$, which is a free energy functional. For an in-depth discussion about this, see Chayes and Panferov (2010). For our proposed dynamical system, we expect the following behaviour:

- a) Weak convergence to equilibrium, e.g., the phase space density tends towards a time-independent measure.
- b) Particles oscillate with smaller and smaller amplitude, but their speed, due to energy conservation, does not reach the zero. Therefore, the particles' behaviour will stand for a continuous function.
- c) Some particles will get accelerated to larger speeds, so that the time-independent limiting phase space density will not have any more compact support.
- d) The particles will display symmetric trajectories.

Summarizing, we require a sort of McKean-Vlasov equation for collective behavior of particles, which takes into account the following (brain) system features:

- 1) A torus, donut-like structure.
- 2) The presence of ions which are at charge equilibrium, e.g., positive charges = negative charges.
- 3) The presence of symmetric and continuous long-range trajectories.
- 4) The knowledge of the density value of the medium embedded into the sphere.

How to test our hypothesis? Previsions. Summarizing the previous paragraphs, both at rest and during spike potentials, the charged particles movements in the brain might be constrained by their electric interactions and

might display long-range plasma-like trajectories. Such collective movements can be described in terms of trajectories along a donut-like torus surface, excluding the central *hole* where intracellular currents take place. How to detect such collisionless spikes movements in real, experimental neurodata? Here the Borsuk-Ulam theorem (BUT) comes into play (Tozzi et al., 2017). Indeed, a less-known version of the BUT states that, given a continuous function, there must be two antipodal points with matching description on a two-dimensional torus (Vendruscolo et al., 2011). The theorem tells us that, if we project a fMRI brain image displaying some activity on a two-dimensional torus, we must find at least two activated regions that are antipodal. This theorem stands just in case the projected function is continuous: this is exactly the case of the hypothesized collisionless brain spikes. Therefore, the above-mentioned BUT variant allows us to evaluate, in fMRI neurodata, two of the required features of the hypothesized collisionless, long-range, plasma-like brain activity: such two features are the occurrence of symmetric activity and a continuous function. This means that, if we are able to find, in fMRI neuroimages projected on a 2-D torus, antipodal regions that are activated at the same time, the brain displays collisionless (plasma-like) activity.

RESULTS

In order to look for the antipodal points and the continuous function able to confirm our hypothesis of plasma-like nervous activity, we retrospectively assessed neurodata available from the literature. In particular, we evaluated fMRI images from Mitra et al. (2015). Their images display high-dimensional spontaneous activity structures, termed “lag threads”, consisting of multiple reproducible temporal sequences. The lag threads were computed from real BOLD resting state rs-fMRI data in a group of 688 subjects, obtained from the Harvard-MGH Brain Genomics Superstruct Project (see Mitra 2015 for further technical details). Twenty-three original video frames (each one in three different projections) from Mitra et al. (2015) (**Figure, left pictures**) were arranged in order to provide the brain with a two-dimensional toroidal structure (**Figure, right pictures**), with the hole centered in the zone of the gray commissure. When the brain is assessed in terms of such donut-like structures, it is easy to see the frequent occurrence of antipodal points with matching activation (asterisks in the **right pictures**). This pattern was found in 18 of the 23 examined frames. In sum, in touch with the hypothesized occurrence of collisionless, long-range trajectories in the brain, our data confirm the presence of both the required ingredients: symmetries and continuous function.

CONCLUSIONS

The occurrence of collisionless, plasma-like movements endowed in the brain currents allows us also to elucidate nervous phase transitions. The brain is sometimes described as a complex, non-linear system, formed by a

large number of interacting and inter-dependent components which exhibit emergent properties, spontaneous self-organization and sensitivity to initial small changes (Bak et al, 1987; Tognoli and Kelso, 2014; Papo, 2014; Afraimovich et al., 2013). It has been proposed that the brain operates at the edge of chaos (Deco and Jirsa, 2012), near a critical regime, where the maximum information function lies between randomness and regularity (de Arcangelis and Herrmann, 2010; Zare and Grigolini, 2013). The treatment of nervous dynamics in terms of collisionless movements on a torus helps in elucidating both the claims that phase transitions occur in brain and their potential mechanisms. Furthermore, it allows us to predict which are the control phase parameters required in order to achieve the predicted nervous criticality. Touboul (2012) analyzed the collective behavior of mean-field limits of large-scale neuronal networks, characterized by a delayed integro-differential McKean–Vlasov equation. The authors identified, through bifurcation analysis, several qualitative transitions in the mean-field limit due to noise.

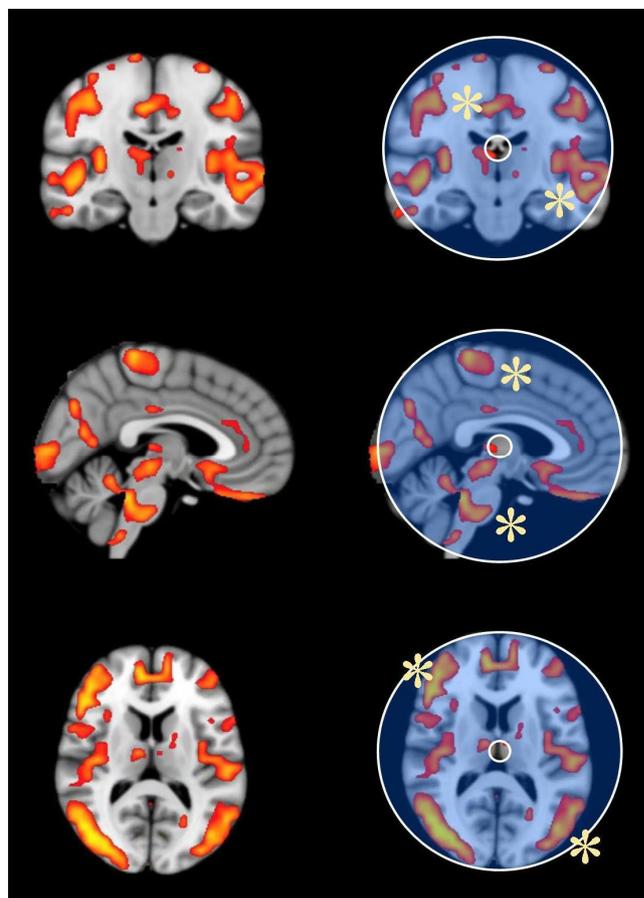


Figure. The left part illustrates three of the 23 assessed video frames, showing brain activities computed from real BOLD resting state rs-fMRI data (Mitra 2015). Three different times and brain projections are displayed. The right part illustrates the images with the superimposition of a two-dimensional donut-like structure. Note the widely diffused presence of antipodal activated regions (asterisks).

In particular, stabilization of spatially homogeneous solutions, synchronized oscillations, bumps, chaotic dynamics, wave or bump splitting were exhibited, arising from static or dynamic Turing–Hopf bifurcations (Touboul, 2012). It means that noise can be used in the evaluation of long-range collisionless nervous trajectories in nonlinear systems. In such a vein, Chayes and Panferov (2010), who studied the McKean–Vlasov equation on the finite tori of length scale L in d -dimensions, were able to derive the necessary and sufficient conditions for the existence of a phase transition (Gates and Penrose 1970). They found indications pointing to critical transitions at the value θ^l of the interaction parameter θ described above. They showed that the uniform density is dynamically stable for $\theta < \theta^l$ and that a *critical* transition occurs at $\theta = \theta^l$. They also showed that, under generic conditions such as L large, $d \geq 2$ and isotropic interactions, the phase transition is discontinuous and occurs at some $\theta_T < \theta^l$. In sum, McKean–Vlasov equations allow us to hypothesize the possible modalities of brain phase transitions and to elucidate their control parameters.

In touch with the old Golgi’s claims concerning the presence of a functional nervous syncytium, recent developments start to unveil that the brain displays a *holistic* behavior (Friston, 2010), acting like a system with long-range interactions. Here we suggest that changes in a small number of brain parameters might lead to long-range nervous interactions. A large repertoire of brain functions and activities (Andrews-Hanna 2014) could be grounded on collective brain phenomena. Indeed, a plasma-like nervous behavior fits very well with the description of the consciousness and might also be responsible of maintaining perceptions, memories and thoughts. This approach, when implemented in a proper lattice or network, might provide a testable alternative to the well-consolidated neural models based on binary logic, such as McCulloch–Pitts neuron and Hopfield networks, or on nonlinear interactions (Tozzi et al. 2016). By an operational point of view, in order to give testable previsions, we need to ask whether it is feasible to use McKean–Vlasov equations in the evaluation of EEG and fMRI patterns of spontaneous and evoked brain activity. It would not take a big effort to compare the nervous trajectories of the real temporal patterns described by previous studies with the theoretical results suggested by collisionless equations.

Furthermore, taking into account trajectories in nervous spaces, we could be able to predict whether they will fall in the donut-like torus suggested by Tozzi and Peters (2016). For example, we speculate that, when particles fall into the torus conformation (where two antipodal particles cannot never meet or become closer, due to their recurrent rotations) the correlated psychical activities could be either repetitive patterns of movements or thoughts, or preserved memories, or mind wandering.

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