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Research article

# The common features of different brain activities

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ABSTRACT

The term "brain activity" refers to a wide range of mental faculties that can be assessed either on anatomical/functional micro-, meso- and macro- spatiotemporal scales of observation, or on intertwined cortical levels with mutual interactions. Our aim is to show that every brain activity encompassed in a given anatomical or functional level necessarily displays a counterpart in others, i.e., they are "dual". "Duality" refers to the case where two seemingly different physical systems turn out to be equivalent. We describe a method, based on novel topological findings, that makes different manifolds (standing for different brain activities) able to scatter, collide and combine, in order that they merge, condense and stitch together in a quantifiable way. We develop a computational tool which explains how, despite their local cortical functional differences, all mental processes, from perception to emotions, from cognition to mind wandering, may be reduced to a single, general brain activity that takes place in dimensions higher than the classical three-dimensional plus time. This framework permits a topological duality among different brain activities and neuro-techniques, because it holds for all the types of spatio-temporal nervous functions, independent of their cortical location, inter- and intra-level relationships, strength, magnitude and boundaries.

## 1. Introduction

The brain activity has been classically divided into different functions. Philosophers (e.g., Kant and Locke) viewed the brain activity in terms of sensations, reasoning, intelligence. Similarly, in current neuroscience, the rather general term "brain activity" stands for a large repertoire of brain functions and mental faculties, such as attention and perception, emotions and cognition, memory and learning, higher cognitive processes (decision making, goal-directed choice, etc.) [1], mind wandering [2]. Many neuro-techniques have been developed throughout the years to assess brain activity at different levels of spatio-temporal observation and evaluate the mental correlates of nervous functions [3]. Every brain activity and experimental tool stands for an observational domain of the whole neuro-scientific discipline, able to evaluate a specific anatomical or functional scale. Some approaches assess brain activity at gross-grained levels of observation, such as EEG and lesion studies [4,5]. Others consider a meso-level of observation, e.g., localized brain areas and sub-areas, such as diffuse tensor imaging, MEG analysis and fMRI resting state functional connectivity [6]. Further approaches allow the assessment of more coarse-grained levels, e.g., microcolumns [7], or single-neuron function and structure Tozzi [8]. More reductionist approaches focus on the molecular levels of brain activity: see, for example, Jacobs et al. [9], Ekstrand et al. [10]. Other techniques favor an approach involving many functional and anatomical levels, tackling the issue of brain functions in terms of non-boundary wall domains spanning over every observational dimension and scale [11,12], so that far apart levels interact with each other (Touboul, 2012). Consciousness, for example, does not seem to be confined to a single level [13].

We explore the possibility to assess brain activities and their dimensional scales in terms of algebraic topology. We evaluate whether different mental faculties can be reduced to a more general one, and whether the division of mental faculties (e.g., the spit cognition/emotion) holds true. Topological concepts make it possible to achieve generalizations that allow the mathematical and empirical assessment of every possible brain activity, independent on its scale, magnitude, specific features and local boundaries. Each brain activity endowed in an anatomical or functional level necessarily displays a counterpart in other ones. This leads to novel scenarios, where different brain activities collide, combine and merge together in a testable way. Therefore, all the brain activities and neuro-techniques are *dual* under topological transformation. In physics, the term "dual" refers to a situation where two seemingly different phenomena turn out to be equivalent: the first can be transformed into the second, so that one ends up looking just like

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the other [14].

## 2. Materials and methods

We provide the conceptual framework to assess brain activities in terms of topological shapes that move in multidimensional abstract environments.

### 2.1. Previous theories

Brain/mind relationships have been tackled by different standpoints, from philosophy – Averroes' unity of intellect, Autrecourt's many intellects (1340), Descartes' duality; Avenarius' empiriocriticism (1907); Dennett's Pandemonium (1991) - to psychology (Ibáñez et al., 2016) and neuroscience [15]. Current neuroscientists split the brain activity in different subsets of mental faculties [16]. Their experimental procedures aim to assess specific observational domains of mental faculties, looking for neural correlates of interacting mental functions [3]. Therefore, neuroscience relies on the tenet that the mind is a functional state of the brain and a clear cortical subdivision among different mental faculties does exist.

Nevertheless, there is no universally agreed definition of what brain/mind's distinguishing properties are. In cognitive neuroscience, "mind" refers to a set of "cognitive" faculties [1,17] i.e., the mental functions which give rise to information processing and embrace: consciousness, perception, attention, different types of memory, language, learning, thinking, judgement, action, attitudes and interaction in the physical, material, social, cultural world [18,19]. "Emotional" states (such as joy, fear, love, hate and so on) [20], alleged to be primitive and subjective, are not encompassed in the definition of the mind as such. Other scientists favor a more general definition of mind, which includes all mental faculties [21]. They argue that rational/emotional states cannot be separated, because, being of the same nature and origin, they should be considered all part of it as mind. We will describe how, despite the local cortical functional differences, mental processes can be reduced to a single, general brain activity.

### 2.2. Brain activities as moving strings

Brain activities taking place in nervous physical spaces can be assessed in terms of geometric structures, i.e., regions, areas or shapes embedded on the abstract geometric spaces' surface. Every nervous activity is modelled as a series of paths followed by particles traveling through brain's micro-, meso- or macro- areas. Brain activities taking place at lower-levels of observation (e.g., the neuronal level) can be depicted as structures equipped with n-dimensions, while brain activities taking place at higher-levels of observation (e.g., EEG spikes) can be depicted as $n+1$ -dimensional structures.

We term a geometric structure given by a single brain activity: "string" [22], which stands for a region on the surface of both the real cortical physical spaces and the abstract geometric ones. Because a string is the path followed by a particle moving through a single or many brain areas, its shape might display either zero or non-zero width, bounded or unbounded length. The brain is a collection of tiny regions of space, which are vibrant "strings" in ambient space time. A brain can be depicted as a collection of nervous activities in 3 + 1 space-time (an *n*-dimensional normed linear space) able to project to higher-dimensional functional spaces (an $n+1$-dimensional space). Continuous mappings from lower- to higher-dimensional structures lead to the Borsuk-Ulam theorem ([23,24]).

### 2.3. Borsuk-Ulam theorem (BUT)

BUT states that a single point on a circumference maps to two antipodal (diametrically opposite) points on a sphere. If we consider antipodal nervous activities confined to a single brain area (i.e., strings defined by the paths followed by cortical currents travelling through a 3 + 1 spacetime) instead of antipodal *points*, BUT leads to a region-based brain geometry ([25,26]), because, in a point-free geometry, regions replace points as the primitives [27,28]. Antipodal regions may also stand for the signals detected by different neuro-techniques, such as spatial/temporal patterns, trajectories, entropies, perceived shape boundary intensities ([28,29]). Hence, BUT provides a way to evaluate changes in information among different anatomical/functional brain levels in a topological space, distinguished from purely anatomical, functional or thermodynamic perspectives. A wide range of brain features (such as intraneuronal dynamics, EEG frequencies, BOLD activity) can be described in terms of antipodal geometric structures (strings) on $n+1$–dimensional structures. Brain signals of different scales and types can be compared, because the two antipodal points assessed at higher-dimensional scales of observation (e.g., EEG macro-levels) can be pulled back to single points at lower-dimensional scales (e.g., histological micro-levels) [26]. The two regions (or activities) do not need necessarily to be antipodal [29]. BUT can also assess non-antipodal features, provided there are a pair of regions, either adjacent or far apart, with the same feature vector ([30,31]). Even though BUT was originally described just for convex spheres, it is also possible to look for antipodal points on other structures [32]: whether a brain activity displays concave, convex or flat geometry, it does not matter, because we may always find the points with matching description predicted by BUT. A cortical projection $\pi(A, B) = E$ for A,B,E $\in \{collection\text{ of cortical regions}\}$ is a mapping of a pair of cortical regions *A,B* to a single cortical region *E* [33,34].

The whole brain can be termed a "*worldsheet*" if every one of its subregions contains at least one string (a brain activity) [35]. In topological terms, "worldsheet" designates a nonempty region of a space completely covered by strings, in which every member is a string ([30,31]). A worldsheet is a special case of a world-volume traced by brain activities (strings) in the cortical 3 + 1 space-time. A brain activity occurs on a curved Riemannian surface of spatial dimension $p$ in a cortical space-time of dimension $p + 1$.

### 2.4. Increase in brain functional dimensions

Brain theories are formulated in 4 space-time dimensions, where each point is defined by three space coordinates plus time [36]. The classical view of the 3D brain states that all anatomical and functional nervous constituents are bound to a worldsheet that can move within a 4-dimensional space (3 + 1 dimensional space-time). A distinctive feature of brain activities (strings) is their world-volume actions. For example, the complete action $A_{M2}$ of a brain activity is defined with respect to the following:

$A_{M2}$: kinetic action,

$q = \pm 1$: electric charge of the brain activity,

$A^{[3]}$: 3-form gauge field,

$W_3$: world volume in 3-dimensional Euclidean space, so that $A_{M2} = A^{kin} \int_{W_3} A^{[3]}$.

This classical scheme has been recently "overthrown" by Peters et al. [37], who demonstrated that the total brain functional activity lies in dimension higher than the usual 3D (plus time) space. Functional increases in brain dimensions give rise to interesting consequences. Although strings embedded in the same wordsheet (i.e., brain activities embedded in the whole brain) are antipodal and share matching description, there is however a difference among the strings embedded in different brain areas. The higher the wordsheet's dimension, the more the information encompassed in strings on the same region, because the number of coordinates is higher. Because strings contain more information than their projections in lower dimensions, a region- and string-based BUT make it possible to evaluate systems features in higher-dimensions, leading to an increase in the amount of detectable information. For example, when you watch at a 3D cat embedded in a

3D plus time environment, your brain perceives a higher dimensional cat, encompassing not just the 3D image, but also its emotional content, the feline species, and so on. In turn, dropping down a dimension means that each region in the lower-dimensional space is simpler. Hence, BUT provides a way to evaluate changes of information among different brain activities in a topological fashion, in addition to the standard mathematical (Maartens, 2010 [35];) and thermodynamic [38] approaches.

### 2.5. Brouwer's fixed point theorem (FPT)

FPT states that every continuous function from a *n*-sphere of every dimension to itself has at least one fixed point [39]. FPT applies, e.g., to any disk-shaped area, where it guarantees the existence of a fixed point, which behaves like a whirlpool attracting moving particles. Su [40] gives a coffee cup illustration: no matter how you slosh the coffee around in a coffee cup, some point is always in the same position that it was before the sloshing began. If you move this point out of its original position, you will eventually move some other point in the sloshing coffee back into its original position. In nervous terms, the shape of brain activity, i.e., a string A (denoted str*A*), is the silhouette (the wiring of string str*A*.).

### 2.6. Wired friend theorem

Every occurrence of a wired friend of a string str*A* with a particular shape with k features on an *n*-dimensional brane hypersphere $S^n$ maps to a fixed description that belongs to a ball-shaped neighborhood in $R^k$.

In neuroscientific terms, we can always find a cortical region containing a single brain function (a string), and every function with a particular shape comes together with another one, termed a *wired friend*. Every occurrence of a wired friend string with a particular shape on the structure $S^n$ maps to a fixed description, *e.g.* to another string that belongs to a manifold (or Euclidean space) with different dimensions. Every wired friend is recognizable by its shape, because the shape of a string is the silhouette of a wired friend string. Therefore, every brain activity (embedded in a higher-dimensional brain macro-level) is the topological description of another brain activity (embedded in a lower dimensional brain micro-level), and vice versa.

In sum, through the topological tools of BUT, FPT and multidimensional approaches, we may state that every brain activity (a string) is the description of another. In the sequel, we will show how the merging of different brain activities, occurring as result of the collision, combination and condensation between single strings [36,41], can be operationalized.

## 3. Results

We describe the computational tools that allow brain activities to be merged and reduced to just one. FPT tells us that different brain activities, when depicted as geometrical shapes, necessarily have at least a feature in common and are continually transforming into new homotopically equivalent ones. They influence each other by scattering and combining, to create bounded regions in the brain. Hence, brain activities can stick together to become *condensed*, e.g., worldsheets, portrayed as a collection of interacting elements of geometrical shapes. Fig. 1, **A, B** describe how to build a worldsheet encompassing brain activities.

### 3.1. The merging of brain activities: homotopy equivalence

Brain activity's shapes deform into another, as a result of the collision of a pair of separate shapes. Let a brain activity be represented in Fig. 1**C**. It evolves over time, as it twists and turns through the outer reaches of another one. Fig. 1**D** illustrates an inkling twisting brain activity appearing in the neighborhood of the first one. The two activities begin interacting, so that the first now has a region of space in common with the second (Fig. 1**E**). In effect, due to the interaction, they are partially stitched together. In Fig. 1**F**, a large area of the total brain space occupied by the first activity is partially absorbed by the second. Therefore, we have the birth of a condensed brain activity.

Recall that a brane (e.g., a brain) is a chunk of matter in the cosmos. Let br*A*, brB denote branes. The two branes br*A*, brB become at first connected via at least one open string (not shown) with ends on the branes (Fig. 1**G**), then a complete condensed brane is formed, with concentric branes having a tooth shape (Fig. 1**H**).

Here follows the mathematical treatment for technical readers. Let $f, g: X \to Y$ be a pair of continuous maps. E.g., let $f(X)$ and $g(X)$ be two brain activities with fixed endpoints (i.e., located in a given brain area). A *homotopy* [42] between $f$ and $g$ is a continuous map $H: X \times [0, 1] \to Y$ so that $H(x, 1) = g(x)$ and $H(x, 0) = f(x)$. The interest is in the possibility of deforming (transforming) one brain activity with a particular shape into another with a different shape. This means that the birth of different brain activities, evolving out of the interaction of initially disjoint branes, is allowed. Let $id_X: X \to X$ denote an identity map defined by $id_X(x) = x$. Similarly, $id_Y: Y \to Y$ is defined by $id_Y(t) = y$. The composition $f \circ g(X)$ is defined by $f \circ g(X) = f(g(X))$. Similarly, $g \circ f(X)$ is defined by $g \circ f(X) = g(f(X))$. The sets $X$ and $Y$ are homotopically equivalent, provided there are continuous maps so that $g \circ f(X) \cong \sim id_X(x)$ and $f \circ g \simeq id_y(y)$. [43]. Homotopically equivalent, evolving brain activity shapes which have the same homotopy type lead to a comparison of activities with seemingly varying shapes and sizes.

### 3.2. Example: homotopically equivalent shapes

A stitching action on a pair of equivalent brain activities is a homotopic mapping that splices them together. Brain activities occuring in branes br*A*, br*B* are connected, provided there is at least one open string *O* (denoted by str*O*) with one extremity of str*O* attached to br*A* and with the other extremity of ***strO*** attached to br*B*. This is a further instance of the duality principle in physics. Open strings are strands ending on brain areas [36]. Strings str***O1,*** str***O2*** have nonempty intersection, provided both strings have extremities attached to the same point on a brain area. That is, one brain activity is the dual of another one, provided the first can be deformed into the second. Let K be a set of open strings and let $2^K$ be the collection of all subsets in K. The brane in Fig. 1**G** is an example of a Edelsbrunner-Harer nerve [43], which is a collection $2^K$ such that all nonempty subcollections of Nrv*K* have a non-void common intersection, i.e., $\text{Nrv}K = \{A \in 2^K : \cap A \neq \emptyset\}$. A sample nerve defined by the intersection of open strings str***O1,*** str***O2*** is shown in Fig. 1**I**.

In sum, when the two branes (two brain activities) are transformed into a new one, we have instance of their homotopy of equivalence, where the second brane that has completely absorbed the first. To make an example, the idea of "cat" arises from the perception of many single cats of different size, color, etc. This is an instance of the duality principle in brain theories: one brain activity is the dual of another, provided the perceived cat can be deformed into the imagined one. Therefore, there is no longer any difference between the cat I see and the cat I imagine. From what we have observed about BUT, FPT and topological shapes, non-colliding brain activities have at least a few features in common. Examples of such common brain features are: average gradient orientation of brane surface hills (upward slopes) and valleys (downward slopes), persistence (brane lifespan), local brane surface shape (*e.g.*, convex, concave), brane shape as a result of deformation from another shape (see, e.g., tooth shape in Fig. 1**H**), etc.

## 4. Conclusions

We showed how, despite their local cortical functional differences, mental processes can be reduced to a single, general brain activity. Brain activities, equipped either with antipodal or non-antipodal

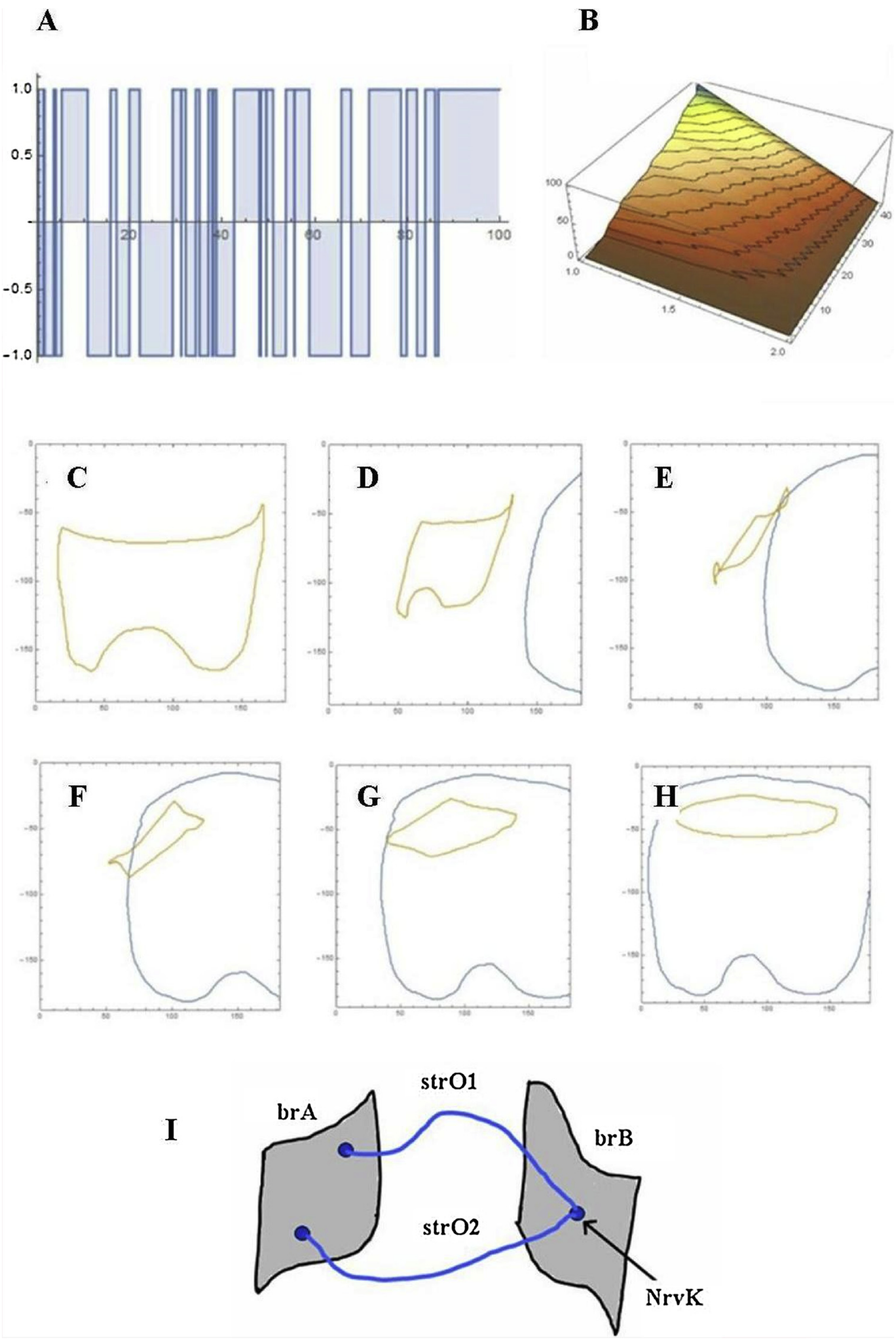


**Fig. 1. A** 2D representation of 40 brain activity collisions over 100 time steps. **B.** A 3D view of a brain activity collision process (0–100 seconds), in which a series of collisions results in their merging [41]. A worldsheet is represented by the 3D surface colliding with brain activities which stand for the vertical walls of the 3D box. **Figures C-H**. Homotopically equivalent shapes, standing for brain activities. A brain activity deform into another, as a result of the collision of a pair of separate activities. **C**: a brain activity (e.g., an emotion) at a given spatiotemporal level. Such level could be differently coarse-grained, e.g., might stand for every micro-, meso- or macro-level. **D**: two brain activities at two different levels. The second shape might stand, e.g., for a cognitive ability. **E**: interacting brain activities. **F**: dual brain activities. **G**: concentric brain activities. **H**: condensed brain activity: emotion and cognition have been merged. Fig. **I**. A set K of open strings strO1, strO2 have extremities on brain activities brA and br B. The extremities of strO1, strO2 meet in a common point on brain activity brB. Hence, nerve.

matching description and embedded in higher-dimensional nervous structures, map to a single activity in lower-dimensional ones, and vice versa. This means that there exists an assessable and quantifiable correspondence between the brain micro-, meso- and macro-levels assessed through a wide range of different neuro-techniques. Our framework holds for all the types of approaches to the brain, independent of their peculiar features, resolution, magnitude and boundaries.

The question is: what for? What are the real-world consequences of the duality among brain activities, by neuroscientists' viewpoint? The first consequence is that the distinction among different coarse-grained levels of nervous activity does not count anymore, because nervous function at small/medium/large scales of neural observation turn out to be topologically equivalent. This generalization points towards an assessable and quantifiable correspondence between different mental faculties. Our common-sense is used to conceive mind faculties as too far apart ever to communicate with one another, so that activities bounded on distant brain regions would never have direct contact: e.g., sensations and abstraction have seemingly very few in common. However, our topological investigation reveals that this belief is unfeasible, because there must be at least one element in common among brain activities that are apparently very distant from each other. Why does our mind separate brain activity in different mental faculties? The answer is straightforward, if we consider topological arguments. If we depict different mental faculties as abstract geometric shapes taking place in the phase space our physical brain, they necessarily display at least a feature in common. Brain signals from two different mental faculties can be compared, because their shapes can be assessed together at higher-dimensional scales of observation. In the brain, every sub-region encompasses at least one mental faculty, which can be modeled as a shape in a topological space. In BUT terms, not only we find a brain region containing a mental faculty, but we also find a mental faculty, embedded in a cortical area, which is the topological description of another one, embedded in another area. In topological terms, mental faculties/shapes are continually transforming into new equivalent mental faculties/shapes. Cortical activities will always have some element in common: they do not exist in isolation, rather they are part of a large interconnected whole. Whether you experience pain or pleasure, or chomp on an apple, or compute a mathematical expression, or quote a proverb, or remember your childhood, or read Wittgenstein's *Tractatus*, it does not matter: the large repertoire of your brain functions can be described through the same topological apparatus.

The second consequence is that mental faculties, lying in multidimensional phase spaces, are dual. Tozzi et al. [44] described how lower-dimensional features can be assessed in the generic terms of particle trajectories traveling on higher-dimensional structures. In our brain scenario, particle movements are operationalized as functions occurring on structures equipped with different geometric curvatures. This methodological advance encompasses diverse brain models that claim for either concave, convex or flat nervous phase spaces ([45–47]). This means that the most of the brain theories are DUAL: e.g., their topological description is the same, despite the huge differences in the subtending manifolds and mechanisms. This useful simplification suggests that the level of observation is not significant when evaluating brain activities, because such levels are fully interchangeable. Because projections among dimensions describe neural phenomena spanning from the smallest to the highest scales, the distinction among different coarse-grained scales does not count anymore: nervous activity is topologically the same at small, medium and large scales of observation. This means, for example, that different types of cognitive studies assess the same topological activity, regardless of their dissimilar protocols and procedures.

Multichannel recordings reveal travelling waves of neural activity in multiple sensory, motor and cognitive systems at single-area (mesoscopic) and whole-brain (macroscopic) scales [48]. Such waves, both spontaneously generated by recurrent circuits and evoked by external stimuli, travel along brain networks at multiple scales, transiently modulating spiking and excitability as they pass. An appropriate projection mapping shows that, if two regions display matching features (e.g., the same intensity, or length, or pairwise entropy) [49], they stand for the same activity. Activities with matching ends (regions) in diverse cortical areas might also help to throw a bridge between spontaneous and evoked brain activities, which become just two sides of the same coin, made of topologically-bounded dynamics. Also, our topological account allows a characterization of the elusive phenomenon of "consciousness": the latter could topologically stand for the condensed brain activity that takes place in the highest-dimensional functional brain levels. In touch with of Cusa's [50] and Bruno's [51] claims, a sort of "coincidentia oppositorum" (our matching description?) occurs among the brain activities endowed in our skull, giving rise to consciousness. The tight coupling among different neural activities gives rise to brains that are in charge of receiving and interpreting signals from other cortical zones, in closely intertwined relationships at every spatio-temporal level. Topology becomes one of the central information processing strategies of the nervous system and allows us to achieve a general, testable, computational model of brain activity that keeps into account the functional subdivisions of mental functions.

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