

Chapter 2

GAUGE FIELDS IN THE CENTRAL NERVOUS SYSTEM

Arturo Tozzi

Center for Nonlinear Science, University of North Texas
1155 Union Circle, #311427
Denton, TX 76203-5017 USA
Computational Intelligence Laboratory, University of Manitoba
Winnipeg R3T 5V6 Manitoba

Biswa Sengupta

Department of Engineering (Information Engineering)
University of Cambridge
Cambridge, United Kingdom

James F. Peters

Department of Electrical and Computer Engineering,
University of Manitoba
75A Chancellor's Circle, Winnipeg, MB R3T 5V6 Canada and
Mathematics Department, Adiyaman University 02040
Adiyaman, Turkey

Karl J. Friston

Wellcome Trust Centre for Neuroimaging, University College
London, London, United Kingdom

ABSTRACT

Recent advances in neuroscience highlight the complexity of the central nervous system (CNS) and call for general, multidisciplinary theoretical approaches. The aim of this chapter is to assess highly organized biological systems, in particular the CNS, via the physical and mathematical procedures of gauge theory – and to provide quantitative methods for experimental assessment. We first describe the nature of a gauge theory in physics, in a language addressed to an interdisciplinary audience. Then we examine the possibility that brain activity is driven by one or more continuous forces, called *gauge fields*, originating inside or outside the CNS. In particular, we go through the idea of *symmetries*, which is the cornerstone of gauge theories, and illustrate examples of possible gauge fields in the CNS. A deeper knowledge of gauge theories may lead to novel approaches to (self) organized biological systems, improve our understanding of brain activity and disease, and pave the way to innovative therapeutic interventions.

INTRODUCTION

Current technical advances in systems neuroscience (Dayan et al., 2013; Mattei, 2014; Huth et al., 2016; Taylor et al., 2016) try to establish causal relations between specific aspects of neuronal activity and system-level consequences. Multidisciplinary theoretical approaches, which offer a bridge connecting the scientific languages of biology, mathematics and physics, are clearly required. One might wonder if the unpredictable complexity of biology – neuroscience included – could hope for that kind of theories. An affirmative answer results from a treatment of living systems – and in particular the central nervous system (CNS) – via the physical and mathematical procedures of a gauge theory approach.

A possible role for gauge fields (Sengupta et al., 2016a) and symmetries (Tozzi and Peters 2016a) has been recently proposed in order to elucidate physiological and pathological features of brain activity. In the search for a dynamic interplay, a cross-over of the physics of elementary particles and cortical brain dynamics, gauge theory takes into account the possibility that brain activity is driven by one or more continuous forces represented by gauge fields, originating inside or outside the CNS. Three main ingredients are required to sketch gauge theory of the CNS:

- A system equipped with a symmetry and a correlated, measurable Lagrangian
- A continuous Lie group of local forces (transformations) which break the symmetry locally.
- One or more gauge fields, (possibly) external to the system, able to restore the broken symmetry and to keep the Lagrangian invariant, despite the local transformations.

The importance of such an approach rests upon the implicit ability to measure unknown quantities: if we knew two of the ingredients (for example, the values of the Lagrangian and the total forces applied to the system), we can extrapolate and calculate the value of the third (for example, the gauge field), via procedures from differential geometry. Indeed, vector projections can be used in order to assess CNS diseases in the powerful probabilistic framework of gradient descent trajectories along manifolds equipped with negative curvatures (Sengupta et al., 2016b).

The three above mentioned ingredients are interchangeable. System, local forces and gauge field may indeed play different

roles in the CNS, depending on our initial assignment. Sengupta et al. (2016a, 2016b) framed the required ingredients in the context of the free-energy principle and focused on the methodological implications of gauge theory for parameterisation and new inference schemes in data analysis. There are, however, other neurobiological possibilities, that speak to a neuronal gauge theory. It is conceivable that brain dynamics constitute more than one symmetry breaking and more than the expression of a single gauge field. A complex interaction among many actors might take place in the CNS, each with its specific role in preserving more than one Lagrangian – and our cognitive functions and associated pathologies could be the result of a mixture of many functional elements.

This paper comprises six sections, and extends neuronal gauge theories for brain function to consider several scenarios. The first section informally describes a gauge theory in physics. We address this introduction to a broad interdisciplinary audience and try to make it accessible to experts from different fields. The second section focuses on the features required for a gauge theory for the CNS; e.g., vector spaces and symmetries, while the third is an effort to describe what gauge theories bring on the table – in the evaluation of brain function. Section four provides the mathematical formalism for technical readers. In section five, we provide gauge theories within the framework of the free-energy principle. Finally, section six considers about other possible biological or functional candidates for brain symmetries/gauge fields.

1) WHAT IS A GAUGE THEORY?

A gauge theory is a field theory, in which the Lagrangian (a function that summarizes the dynamics of the system) is invariant under a continuous group of local transformations (Zeidler, 2011). The most important physical theories of the last centuries; i.e., electromagnetism, general relativity and quantum field theory, can be framed in gauge theories. The underlying concept is quite simple: *gauge* means *choice*. A *gauge* is nothing more than a *coordinate system* that varies depending on one's *location* with respect to some *base space*.

The cornerstones of gauge theories are the concept of *symmetries* and the Noether theorem, which states that for every continuous symmetry there is a conserved physical quantity. The global symmetry of the system is preserved, in spite of local changes, by a continuous force, called the *gauge field*. A gauge theory can be studied via normalized

mathematical procedures (t'Hooft, 1971), so that the local forces acting on each point of the system can be quantified and investigated through differential geometry. We will try to explain what a gauge theory is, conveying the geometric intuition rather than the rigorous formalism.

To illustrate the formalism, we might start with a ball, representing a system. Local forces, depicted as vectors, act just on a few zones of the ball surface (**figure 1A**). Indeed, vector spaces have noteworthy properties that make them attractive for representation models and for encoding complex structures as single multidimensional vectors (Snaider and Franklin, 2014). The ball is unfolded and flattened into a two-dimensional reconstruction, allowing the entire surface to be transferred to a bidimensional *circle* (**figure 1B**). The local forces are arbitrarily described as vectors of different lengths and orientations, originating from the circle surface (**figure 1B**). The local forces on the circle surface can be described in terms of a broken line (the dotted line in **figure 1C**). In this case, the symmetry is considered as locally *broken* or, better, *hidden*. In order to keep the global symmetry invariant, we need to introduce a balancing force (**figure 1D**). This force is the *gauge field*. When the gauge field comes into play, we obtain a system equipped with continuous and unbroken line (the dotted line parallel to the circle surface in **figure 1E**). This line stands for the Lagrangian, roughly corresponding to the global symmetry. In summary, if we want to keep the Lagrangian invariant, despite local changes, we have to transform the broken line of **figure 1C** into the continuous line of **figure 1E**, with the aid of a counteracting force. When a system preserves its own symmetry despite the action of local forces, it is said to be equipped with a gauge symmetry and we have a gauge theory.

When one sketches a gauge theory, one can arbitrarily choose a symmetry *a priori*: by *fixing a gauge*, the model then becomes easier to analyze mathematically. However, this does not automatically mean that every hypothesized gauge theory should be accepted as valid. Deciding exactly how to fix a gauge is a key issue: the tractability of the resulting problem is heavily dependent on the choice. Although gauge theories approach their task speculatively, they build entirely upon the results of the physical sciences. They must be not only logically, but also physically tenable. In sum, a gauge theory is an abstract conjecture that needs to be tested by empirical investigations. As an example, the Higgs boson was first hypothesized via a gauge symmetry (Higgs, 1964), then confirmed with the Large Hadron Collider.

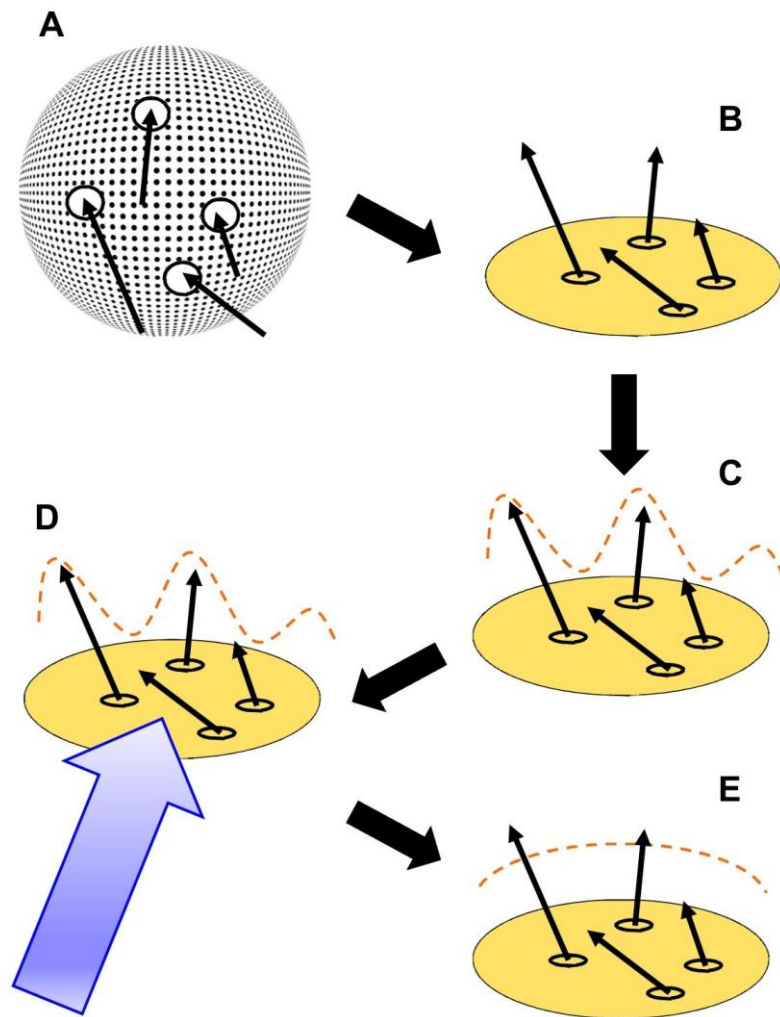


Figure 1. A geometrical illustration of a gauge theory. The gauge field, a sort of mathematical *deus ex machina*, balances local transformations. See text for further details.

2) IN SEARCH FOR SYMMETRIES IN THE CNS

In this section, we focus on the key problem in sketching a gauge theory for the CNS. To do this, we need a continuous symmetry to break and restore. The search for nervous symmetries is not easy. Due to our incomplete knowledge of brain function, we do not know exactly which, and how many, symmetries are hidden in the CNS. Furthermore, symmetries need to be constrained and it is unrealistic to seek – in biology – the mathematical simplicity of physics. Even if we knew the requisite symmetries, it is doubtful if our current technology would be able to calculate all the necessary variables at each point in the CNS. For example, recent studies suggest that

cognitive functions do not depend solely on electrical pulses, but on multifactorial intra- and extra-neuronal causes, involving supramolecular interactions among biologically active macromolecules (Tozzi, 2015). It is also important to remember that any hypothetical descriptions cannot unambiguously characterize the etiology of fluctuation properties, as similar symmetric properties may stem from qualitatively different generators, which may be difficult to distinguish with finite data (Papo 2014).

Despite these limitations, gauge theories could pave the way to a novel approach to organized living systems. The prevalence of complex fluctuations would allow not only treating the brain as a physical system, but also help classify cognitive processes as operators acting on symmetries. Moreover, the computational neuroscience community is currently undertaking an effort to provide a systematic way to characterize symmetry and asymmetry in the network structures of the connectome – by inspecting the eigenvalues of different types of connectivity matrices (Esposito et al., 2014). Gauge theories for the brain are reminiscent of a mechanism of homeostasis, in which: a) the conserved variable is a symmetry, often hidden from our observation, and b) the *balancing* force must be continuous. In what follows, we present a brief list of known symmetries in CNS that could be tested in the frame of a gauge theory. The following treatment recalls recent results by Tozzi and Peters (2016ab).

The brain generates scale-free fluctuation, even in the absence of exogenous perturbations (Papo 2014). It has been suggested that scaling properties allow cognitive processes to be framed in terms of complex but generic properties of brain activity at rest and, ultimately, during functional operations, limiting distributions, symmetries, and possibly their universality classes (Papo, 2014). The stability of spectrum exponents (of many neuronal processes) suggests that a universal scaling characterizes a large class of brain systems and physiological activities (de Arcangelis 2012). Complex scaling and intermittency are generic spatiotemporal properties of the brain and, more importantly, could contain information on how observable large-scale behaviours arise from the interactions of many small-scale processes (Papo 2014).

The $1/f^n$ structure – of time evolving neuronal activity – offers a parameterisation that summarizes the values of the frequency and amplitude of cerebral activity over a given time. This means that a high number of possible source configurations of brain currents, in different cortical areas (i.e., *local transformations*, such as gamma oscillations in somatomotor

cortex during states of enhanced vigilance, or alpha waves in posterior zones with the eyes open) give rise to a set of measured potentials characterized by a general scaling property. In a gauge theory, the global invariant power law might stand for the complete Lagrangian density of the total brain currents – and can therefore be regarded as a global symmetry; while the local changes in frequency and amplitude are the local gauge symmetries. In summary, if we knew the frequencies and amplitudes of activity at each point of the brain, we could be able to estimate frequency and amplitude of the required, hypothetical gauge field.

Another neuronal symmetry has been recently discovered. A constant excitatory/inhibitory (E/I) ratio between the total amount of excitatory and inhibitory stimulation has been described, both *in vitro* and *in vivo* (Haider, 2006). The balance between the two opposing forces affects many cortical functions, such as feature selectivity and gain (Xue) and memory of past activity (Lombardi). The E/I ratio could also be interpreted as evidence of a homeostatic mechanism between strengthening and weakening processes in the adaptation of synaptic neuronal connections (de Arcangelis 2012). Homeostatic systems induce a distinction between inhibitory and excitatory connections that could contribute to symmetry breaking, leading to directed coupling and information transfer (Tognoli and Kelso, 2014). From our point of view, E/I ratio might be regarded as the Lagrangian of the intact and spontaneously active cerebral cortex. The experimental evaluation of local symmetry breakings in cerebral cortex could provide a map of the continuous forces acting on brain, in order that the gauge field could be calculated via differential geometry.

Thus far, gauge theories of CNS have been already proposed in the framework of quantum mind theories (Freeman and Vitiello, 2008; Matsui, 2001). Some of these models are equipped with a local gauge symmetry and resemble lattice gauge theory of high-energy physics. They are grounded on the notion of spontaneous symmetry breaking (Freeman and Vitiello, 2008). The symmetry that is broken is the rotational symmetry of the electric dipole of the vibrational field of water molecules. Environmental stimuli may therefore act as a trigger for the breakdown of such symmetry.

It has been suggested that the cerebral cortex exhibits a fairly uniform microarchitecture, e.g., the minicolumns, characterized by a modular connectivity with invariant properties (Casanova et al., 2011). Specifically, minicolumns exhibit a translational symmetry across their central axis and rotational symmetry;

i.e., displacement in different planes of section. Furthermore, they are equipped with transitive symmetry, with respect to geometric scaling of morphometric relations in different cortical areas, and with temporal symmetry of morphometric relations during cortical maturation. Evaluation of how architectonic relations among minicolumnar elements (e.g., pyramidal cells) are conserved under spatial and temporal variation might lead to a better understanding of diseases characterized by columnar anomalies, such as autism, schizophrenia, Alzheimer's and drug addiction (Opris and Casanova, 2014).

Last, but not the least, the free energy principle (FEP) for adaptive biotic systems (Friston, 2010) might be regarded as another symmetry hidden in the CNS (Sengupta et al., 2016). This is the most important candidate, because it encompasses all the above symmetries into a very general framework. Given its broad explanatory scope, we will return to the FEP later, in a dedicated section.

3) GAUGE THEORIES: WHAT ARE THEY GOOD FOR?

Gauge theories originate from physics. However, they could, in principle, be applied to countless fields of biology, such as cell structure, bodily physiology, and so on. In this chapter, we focus on the CNS. The first and most important question is the following: is it possible to transfer powerful gauge symmetries from their natural environment of physical particles to the *soft* (and much more complex) living structures of biology? In other words, are we allowed to sketch a gauge theory of brain function?

The importance of a gauge theory calls on the possibility of applying differential geometry to CNS activity, in order to quantify unknown, hidden or latent variables. The possibility of using vector spaces and geometrical structures instead of neurons and wires for a representation model of the CNS has been already explored. To give an example, a modular small-world topology in functional and anatomical cortical networks has been shown considered as an information processing architecture (Jarman). Furthermore, researchers have demonstrated that visual experience is two-scaled, with a smaller dimension at shorter length scales and another at longer scales (Sreekumar). Moreover, it has been proposed that entorhinal grid cells reflect a stable 2-dimensional manifold that contains the activity of individual neuronal representations, driven by continuous attractors (Yoon et al., 2013).

A central hypothesis, in this context, is that the information encoded in place-cell replay should reflect the topological structure of the experienced surroundings, thus capturing the spatial complexity of our environment (Wu and Foster, 2014). In order to use vectorial models in the assessment of the brain, the spatial characteristics of neuronal connectivity need to be considered. For this purpose, some scientists endow networks with a metric by embedding them into a physical space. This provides an adaptive rewiring model with a spatial distance function and a corresponding spatially adaptive (local) rewiring bias, which predicts observed connectivity architectures (Jarman). Further, by linking notions from Lagrangian and Hamiltonian mechanics of rigid bodies, some investigators have defined human shape as a Riemannian metric space, generalizing D’Arcy Thompson’s classical formulation of mathematical morphology of shape and form, with the metric structure defined by the geodesic flow of coordinates connecting one shape to another (Djamanakova).

Software schemes offer tools for integrating structural and functional information across anatomical scales, thus connecting information across multiple physiological scales (Djamanakova et al., 2014). Also various neuro-epistemological approaches are based on vectors. This idea was first put forward by Alfred North Whitehead (Whitehead, 1919) and Kurt Lewin (Lewin, 1935) and then pursued, among others, by eliminative materialism (Churchland) and integration information theory (Tononi, 2007).

In common with the above, gauge theories also allow the projection from a *real*, external space onto an *abstract*, more manageable space equipped with sufficient statistics. However, a gauge framework is much more powerful and accurate (**figure 2**). Gauge fields have a practical advantage: their forces may be exactly calculated through a difficult, but feasible experimental energy-based variational approach. Gauge theories can be evaluated through topological tools such as the Borsuk-Ulam theorem (BUT). BUT states that (Borsuk 1933; Dodson, 1997):

Every continuous map $f : S^n \rightarrow R^n$ must identify a pair of antipodal points (on S^n).

This means that the sphere S^n maps to an n -dimensional, Euclidean space R^n (Tozzi and Peters, 2016b). Points on S^n are *antipodal*, provided they are diametrically opposite. For the use of BUT and its variants, see Peters and Tozzi (2016). It has

been recently proposed that symmetries lie on the S^n sphere in guise of antipodal points, while the broken symmetries lie on the corresponding R^n manifold (Tozzi and Peters, 2016a). Here, the gauge field stands for the continuous function required to go from a level to another (**figure 3**). This approach allows us to assess vector and tensor projections also by using the powerful tools of algebraic topology (Peters 2016).

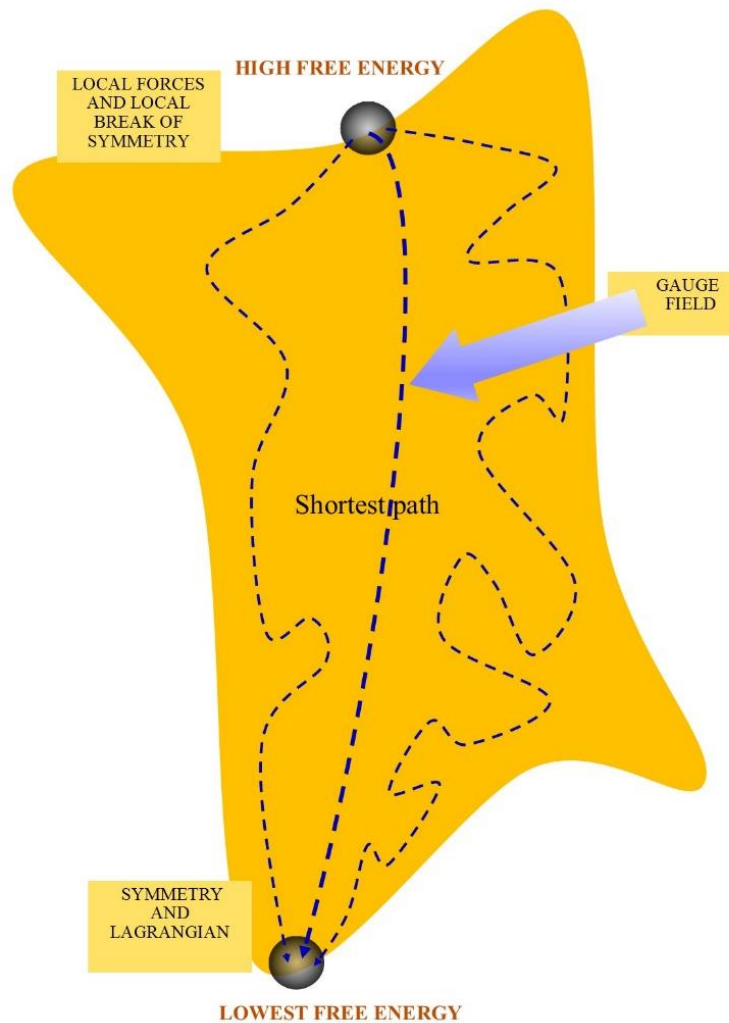


Figure 2. Neuronal phase spaces equipped with sufficient statistics allow a rigorous way of measuring distances on concave manifolds. This means that dynamics transporting one distribution of neuronal activity to another is given by the shortest path from point to higher energetic levels to lower ones. In mathematical terms, the gauge field is a Levi-Civita connection, which allows exponential mapping and parallel transport, following the steps of a Langevin equation. For further details, see Sengupta et al. (2016).

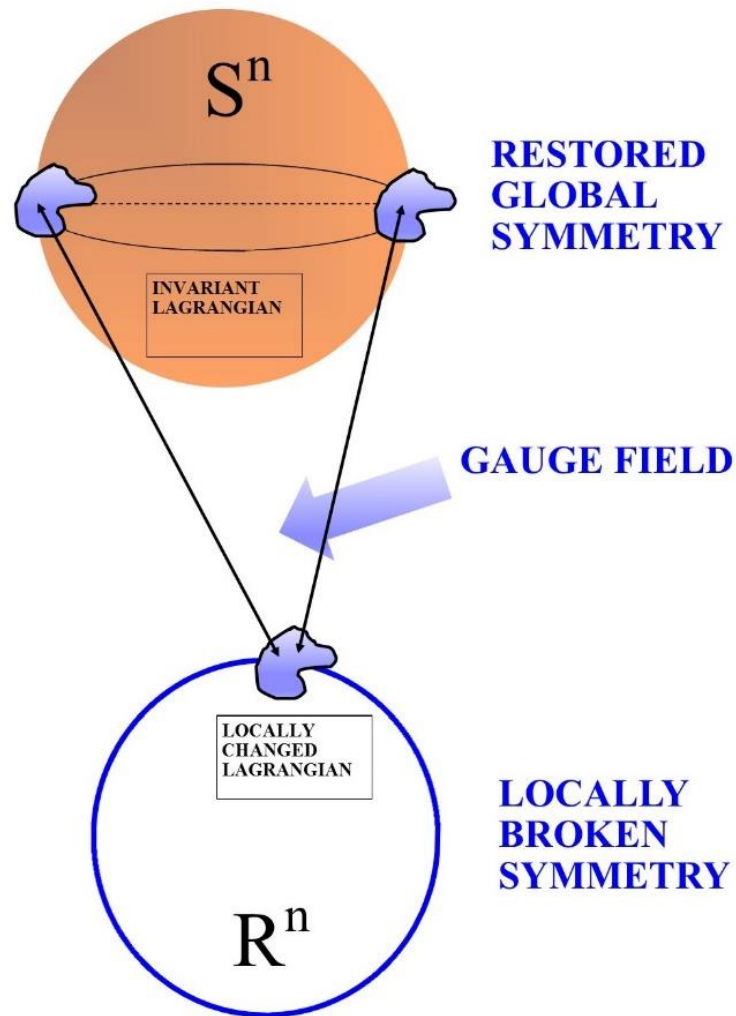


Figure 3. Borsuk-Ulam theorem and gauge fields. BUT and its variants require a function which needs to be continuous. In the same way, a gauge theory requires a gauge field which needs to be continuous.

4) MATHEMATICAL FORMALISM OF A CNS GAUGE THEORY

There are many possible ways to handle a gauge theory of the CNS in a differential geometric sense. Analytically, this is simple but the numerics can be tortuous. Sengupta et al. (2016) used Levi-Civita connections to project from an abstract space to another, more manageable space. Here we will focus instead on another type of transport, e.g., the Ehresmann connection.

As an example, we will consider the hypothetical case of a system of cerebral electric potentials equipped with a global invariant $1/f^n$ scaling symmetry. A procedure to quantitatively assess the required gauge fields is as follows. Neurophysiologic measures of brain activity in humans usually use a simple, non-invasive electroencephalogram (EEG). These measures exhibit significant $1/f$ -like power spectrum scaling (Pritchard, 1992). Subjects would undergo 10-20 EEG measurements during active concentration and rest, eyes open and closed. In **figure 4A**, as an example, we choose four random cerebral areas with hypothetical frequency and amplitude measures of EEG activity. The n value of recorded EEG segments could be estimated by fractal analysis techniques (Ihlen 2012). The most interesting cognitive phenomena occur in time windows shorter than seconds (Buzsáki and Watson, 2012). Consequently, EEG frequencies and amplitudes might be evaluated in sub-seconds frames.

Once acquired, data from each point of the cortex can be investigated using the formalism of differential geometry. Technically speaking, a continuous group of transformations is applied on the tangent bundle, then a local section of the principal bundle is chosen and the covariant derivative is calculated via an Ehresmann connection. Practically, this would involve the following. The cerebral hemispheres (**figure 4A**) are unfolded and flattened into a two-dimensional reconstruction (Van Essen, 2005), allowing the entire cortical surface to be transferred to an atlas \mathbf{M} of \mathbf{C}^K (differentiable), \mathbf{C}^∞ (smooth), finite dimensional manifolds, each one mapping a brain area (**figure 4B**). The set of power spectra describing each cortical area's frequencies and amplitudes now stand for a continuous group of local transformations acting on sections of \mathbf{M} . \mathbf{M} is arbitrarily equipped with a constant matrix \mathbf{G} belonging to the $SO(3)$ Lie group, isomorphic to the rotation group of the sphere (**figure 4B**).

\mathbf{M} is a principal G -bundle \mathbf{P} characterized by a trivial, smooth and differentiable fiber bundle, by vector bundles \mathbf{E} and by a tangent bundle \mathbf{TE} (**figure 4C**). The electrical forces are described by numbers, arranged in vectors and angles, representing the action \mathbf{G} on a chosen local section \mathbf{E} of \mathbf{P} . Four forces \mathbf{G} are depicted in **Figure 4C** in the guise of four vector bundles \mathbf{E} arising from four points \mathbf{p} in the tangent space \mathbf{T}_p . They are equipped with four n -dimensional rotation angles $\boldsymbol{\varphi}(\varphi_1, \varphi_2, \dots, \varphi_n)^T$ standing for the local $1/f^n$ scaling of each of the four brain areas. Rotations through tiny angles link nearby transformations of angles $\boldsymbol{\varphi}$ arising from points \mathbf{p} . As a result, the linear approximation of the function \mathbf{G} at \mathbf{p} (and its angle $\boldsymbol{\varphi}$) in each dimension can be described by introducing a partial

derivative. In brief, changes in degrees of ϕ in selected brain areas match with different power spectra and hence with different electric configurations.

In a gauge theory, the geometric *link* between L and ϕ can be defined in terms of a connection form, the Ehresmann connection (Ehresmann, 1950). If we identify the horizontal space \mathbf{H} , perpendicular to the vertical space \mathbf{VE} , we can extrapolate the Ehresmann connection ω , which is a vector on \mathbf{TE} (**figure 4C**). The Lagrangian density L is indeed a function of \mathbf{TE} and \mathbf{H} . It would be correct to formulate all rates of change of ω and ϕ in terms of covariant derivative, a linear differential operator in each associated \mathbf{TE} , which allows different points (and their angles) to be compared. Mapping every vector ω of \mathbf{P} into the bijective, diffeomorphic \mathbf{P}^I space, enables one to derive a curvature form (**Figure 4D**). When the vectors ω^I intersect the unique horizontal lift – corresponding to the invariant L – the angles σ are obtained.

The behavior of the vectors ω^I and angles σ can be compactly written by point-wise vector addition of the partial derivatives of the function G at each point. As a result, we get a single vector: $\vec{\Omega} = \vec{\omega}_1 + \vec{\omega}_2 + \dots + \vec{\omega}_n$. The angle Σ is introduced (**figure 4E**), standing for the interaction Lagrangian L_{int} and expressing the $1/f^n$ scaling values of vector addition obtained from our experimental procedure. If the lines L and Ω are parallel, Σ equals the zero, L_{int} equals L and the symmetry of the system is preserved. Otherwise, if L and Ω are not parallel (as is usual in physiological systems), Σ departs from zero, L_{int} is different from L and the system displays a broken or absent symmetry. In this case, in order to ensure the invariance of L and to restore the symmetry, we need to define a covariant derivative such that the derivative of Σ will again transform identically with Σ . According to the covariant version of gauge theories, the correction terms are reinterpreted as couplings to an additional divergent counter-term, the gauge field, by allowing the symmetry parameter to vary from place to place in the local coordinate system.

Figure 4F shows the above procedure in a very abridged form. If we ignore L and examine the vector Ω and its angle Σ , we observe nothing else than a single force. If we instead regard L as a vector, whose basis results from the scalar components of its vector space \mathbf{v} , then Ω (and its angle Σ) turns out to be just one of the covariant components of L . In order to keep L invariant, we need to add another component into \mathbf{v} : we introduce the vector Ψ , equipped with the angle Θ (**figure 4F**). The angle Θ stands for the gauge field Lagrangian L_{gf} and expresses the global value of $1/f^n$ scaling of the required gauge

field. We are thus allowed to make accurate predictions of forces: we can extrapolate from Θ the values of time, frequency and amplitude of the gauge field required to keep invariant the $1/f^n$ scaling during cortical activity. This provides a practical example of how one can apply gauge theories operationally to electromagnetic recordings of the sort that are currently available in systems neuroscience.

A mathematical formulation for technical readers and a tutorial are provided in Sengupta et al. (2016), Supporting Information.

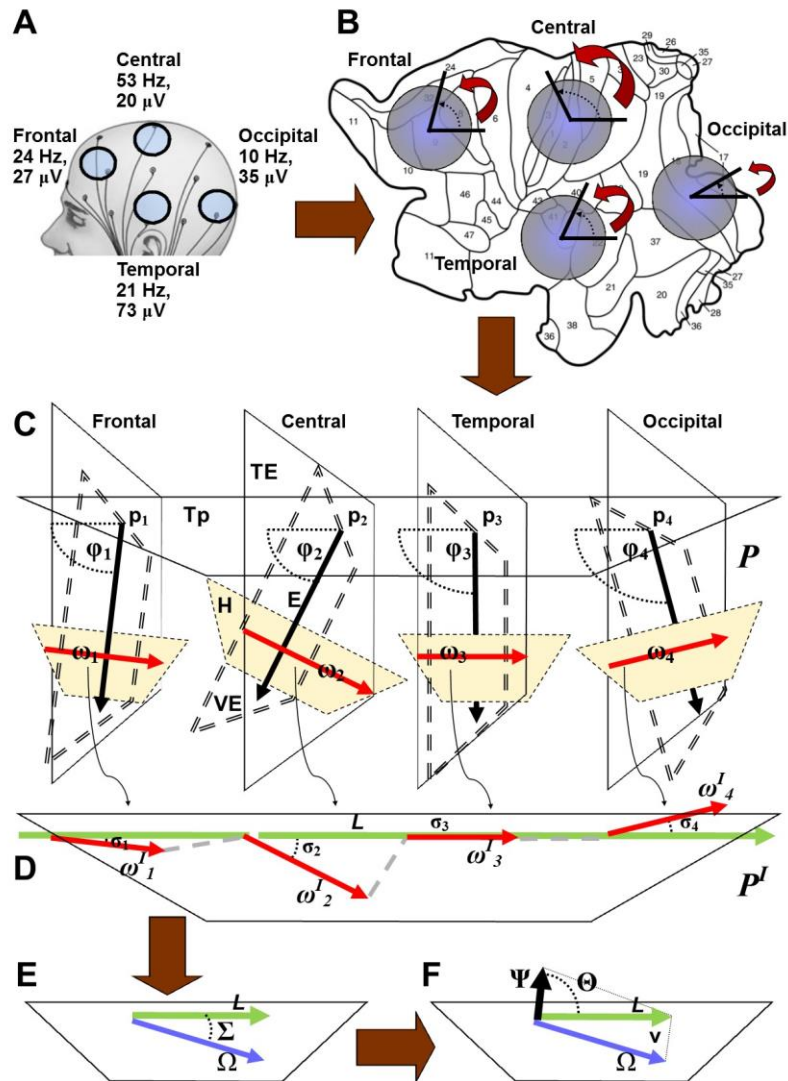


Figure 4. Dynamics of cortical electric fields, described in terms of sections of fiber bundles, jet manifolds, Lie groups and Ehresmann connections. See the main text for further details.

5) GAUGE THEORIES AND THE FREE-ENERGY PRINCIPLE

The free energy framework is the most natural candidate for a gauge theory of the CNS (Sengupta et al, 2016). The free energy principle says that self-organizing system that is at nonequilibrium steady-state with its environment must minimize its free energy, thus resisting a natural tendency to increase its disorder or entropy (Friston 2010). The principle of minimising variational free energy (an information theoretic construct for systems that attain non-equilibrium steady-state), is about trying to understand how homeostasis is maintained (Sengupta 2013). The free energy principle separates the agent (the internal states) from the environment (the external states), which encompasses the external and internal milieu. Because entropy is also the long-term average of *surprise*, the brain will appear to minimise surprise (aka *surprisal* or self information), in order to ensure that the probability of interoceptive and exteroceptive sensory states; i.e., the entropy of sensory exchanges with the environment remains low.

Agents can suppress surprise by changing the two things that surprise depends on: either they modify sensory input by acting on the world, thus minimizing prediction errors, or they adjust their perceptions, by changing their internal states, thus optimizing accuracy of predictions (Friston 2010). The feasibility of calculations based on entropy has been proved useful for a basic understanding of neuronal complexity. Research looking at fluctuations in brain signals provided evidence that the complexity of those signals, as measured by entropy, conveys important information about network dynamics, e.g., local and distributed processing (McDonough). Efficient algorithms to calculate transfer entropy values between two systems have been introduced, allowing the estimation of functional connectivity between different brain regions (Ma). Furthermore, resting-state fMRI data from the Human Connectome Project (Van Essen et al., 2013) were used to measure, through multiscale entropy, the extent of neural complexity in the BOLD signal (McDonough). These studies affirm a prediction of the principle of minimum (variational) free energy that necessarily entails a high degree of mutual information (relative entropy) among distributed representations in neural networks – and a necessary critical slowing of the sort seen in scale free dynamics (Friston et al., 2012).

The time average of variational free energy is essentially a proxy for entropy, therefore minimising entropy production corresponds to minimising variational free energy at each point

in time. Accordingly, we can treat variational free energy as a Lagrangian and, implicitly, a way of minimising entropy. One might imagine that the dynamics associated with this sort of Lagrangian are a necessary consequence of the resulting coupled dynamical system (with a random dynamical contracting set), and a resulting eigen-spectrum with a countably large number of eigenvalues and associated multi-Lorentzian spectral density. As above, tensor values of the required gauge field could be calculated via differential geometry. For example, if the tensors are measured in energy units, we can estimate how much energy (originating from an external or internal gauge field) is required by mental states, in order to minimize entropy and variational free energy at each point in time. **Table 1** recapitulates an account of a gauge theory based on the free-energy principle.

	<i>PHYSICAL AND BIOLOGICAL CONTEXT</i>	<i>VARIATIONAL FREE-ENERGY FORMALISM</i>	<i>POSSIBLE CORRESPONDING NERVOUS STRUCTURES</i>
SYSTEM	CNS	Brain as an inferential machine in non-equilibrium steady-state; an agent with its neuronal or internal states	CNS
LAGRANGIAN	Energetic homeostasis	Variational free-energy (sensory entropy)	CNS homeostasis
LOCAL FORCES	Continuous external stimuli coming from the environment, or internal stimuli coming from the body	External states producing local sensory perturbations in the agent and increase of prediction errors, which gives rise to a Kullback-Leibler divergence between sensation and prediction	Sensory pathways, from the periphery to the CNS, and their corresponding, coupled cortical layers 1-3, which convey bottom-up messages
GAUGE FIELD(S)	External continuous force	Precision weighting of prediction errors (implicit in the encoding of uncertainty), leading to minimization of prediction errors through action and/or perception; Levi-Civita connection, joining together physical spaces and phase spaces	Cortical layers 5-6; they convey top-down messages Visual attention Other unknown structures

Table 1. The ingredients of a gauge theory within the framework of the free-energy principle: See Sengupta et al (2016) for details

6) OTHER CANDIDATES FOR BRAIN GAUGE THEORIES

In summary, to sketch a CNS gauge theory, we require the neuronal homologues of the three ingredients: i.e., a system equipped with a symmetry, local forces and one or more gauge fields. Depending on our initial gauge choice, the three ingredients can be assessed in different combinations. Many possibilities arise, whose feasibility can be experimentally verified. In closing, we provide some discussion of further candidates for gauge fields.

Consciousness. In the search of the neural correlates of consciousness (Koch et al., 2016), gauge theories may provide a novel method to investigate this challenging phenomenon. In particular, when embedded in a gauge framework, consciousness no longer stands for unspecified brain activity, but for a quantifiable parameter that can be expressed in terms of vectors or tensors. Indeed, consciousness might correspond to a Levi-Civita connection; e.g., the gauge field. In absence of consciousness, the external states do not produce sensory perturbations; from the perspective of the free energy principle, this means, the symmetry of the variational free energy, e.g., the sensory entropy, cannot be restored. As an example, during sleep, in which consciousness is altered, a response to local sensory perturbations does not occur. In such a vein, if we assume that the system stands for the cortex, ascending arousal systems are potential candidates for gauge fields. The midbrain contains tonic neurons that ensure a continuous cortical neuromodulation. Tonic neurons are located in the locus coeruleus, in the dorsal raphe and central superior nuclei; e.g., along the pathway of the ventral branch of ascending arousal system, and in the reticular thalamic nucleus, which is the final step of the dorsal branch of the ascending arousal system (Nieuwenhuys et al., 2008). Recent data suggest that the neural correlates of consciousness might be associated with posterior cortical hot zones (Koch et al, 2016). If this was the case, we could be allowed to assimilate qualitative notions of conscious level into the quantitative gauge theoretic framework of cortical function.

Blood flow. If we hypothesize that the system is the entire CNS and the gauge field is located in other bodily systems, it is possible that local forces stand for the cortical oscillations evoked by the stimuli from the external environment, while the continuous gauge field stands for haemodynamic fluctuations. The idea that blood circulation influences brain activity dates

back to the pioneering work of Angelo Mosso (1896) and is still evident in brain mapping studies that predominate in modern neuroscience (Nieuwenhuys et al., 2008; Friston et al, 2014). A recent paper is particularly intriguing in this regard (Park et al., 2014). In humans, cardiovascular fluctuations underlie behaviourally relevant activation in multifunctional cortical areas. Neuronal events locked to heartbeats before stimulus onset predict the detection of a faint visual grating in the posterior right inferior parietal lobule and the ventral anterior cingulate cortex. Heartbeats therefore shape visual conscious experience, by contributing to neuronal representations. Similar coupling in the interoceptive domain would mean that the interoception of autonomic signals might underlie subjectivity and sense of self (Park et al., 2014). Furthermore, emotions can be influenced by cardio-circulatory mechanisms. It has been demonstrated that short-term interoceptive fluctuations enhance perceptual and evaluative processes related to the processing of fear and threat. The processing of brief fear stimuli is selectively gated by their timing in relation to individual heartbeats and these interoceptive signals influence the detection of emotional stimuli at the threshold of conscious awareness, altering emotional judgments of fearful and neutral faces (Garfinkel et al., 2014). Resembling a novel version of Fechner law, gauge theories might quantitatively correlate stimuli and perceptual processing.

Environmental stimuli. In an *autopoietic* account of the gauge framework, the system might stand for the brain, equipped with self-sustained continuous forces producing local, self-generated perturbations. In this case, the best candidate for a gauge field is the continuous afference from the external world. In such a framework, the spontaneous activity of the brain cannot be simply reduced to background noise, uncorrelated to the system's response (Lombardi et al., 2012). We are presented with an auto-referential system, in which the role of spontaneous fluctuations is to preserve brain's internal symmetry. Homeostatic plasticity ensures that neuronal networks assume a sub-critical state, independently of the initial configuration. Surprisingly, increasing the external stimuli modifies the network set-point towards criticality (Priesemann 2015).

Time. It has been recently suggested that some brain functions could be dictated by the principle of minimum frustration, a concept borrowed from energetic landscapes of protein folding (Tozzi et al, 2016). The brain is equipped with many timescales, ranging from nanoseconds to several days. According to this novel formulation, structural changes in the

brain lead to energy decrease at very long CNS timescales. With the passage of time, the trajectories of neuronal processes, such as memory and perceptual recognition, tend towards the low-energy basins of narrow funnel-like attractors. As with the free energy principle, the minimum frustration principle states that the gradient descent (energy decrease) dictated by Langevin equations takes place over long brain timescales (see **Figure 2**). In such a vein, time becomes a gauge field. Indeed, at least at the non-relativistic scales typical of biological systems, time is a continuous function. This function can restore the global symmetry; e.g., the lowest possible variational free energy, which has been broken by local forces, e.g., environmental inputs. Time stands in this case for a known parameter, which affords the possibility to evaluate unknown parameters.

CONCLUSIONS

When emphasizing the circular causality between the nervous system and the world in which it is embodied, a proper consideration of this holistic aspect of information processing in the brain is required. Starting from this conceptual background, our aim was to introduce the general, abstract model of a physical gauge theory in biology and evaluate its possible implications for the CNS. Brain functions and neuropsychiatric diseases can be approached from the physicist's point of view – appealing to the basic observation that the CNS is an open system; continuously interacting with its environment. The ensuing dissipative character of the brain turns out – from a gauge theoretic perspective – to be the root of its dynamics, behaviour and persistence.

Such an approach holds promise because it directs us into new ways of thinking, offers new perspectives, and shows how it is possible to reformulate old problems. Gauge symmetries could lead to novel approaches to organized biological systems, improve our understanding of brain function and pave the way to innovative therapeutic strategies. To take some examples, visual attention, consciousness, spontaneous neuronal activity and functional regimes at the edge of criticality are impaired in many pathological conditions, such as autism, schizophrenia, drug addiction, Alzheimer Disease, depression *etc.* A potential medical application for gauge fields is epilepsy, in which, according to a gauge theory, the Lagrangian might be disrupted by pathological spikes. Seizures could be counteracted, or even removed, by carefully constructed 'artificial gauge fields' (e.g., via selective application of electric waves of specific frequency on target micro-areas, or via drugs) able to *recover* the

Lagrangian and restore the electric symmetry. We hope that this brief survey of the potential of a gauge theoretic approach hints at the possibility of such developments in the future.

BIBLIOGRAPHY

1. Borsuk, M. Drei sätze über die n-dimensionale euklidische sphäre, *Fundamenta Mathematicae* XX (1933), 177–190.
2. Buzsáki, G., Watson, B.O. (2012). Brain rhythms and neural syntax: implications for efficient coding of cognitive content and neuropsychiatric disease. *Dialogues Clin. Neurosci.* 4, 345-367.
3. Casanova MF, El-Baz A, Switala A. Laws of conservation as related to brain growth, aging, and evolution: symmetry of the minicolumn. *Front Neuroanat.* 2011 Dec 26;5:66. doi: 10.3389/fnana.2011.00066. eCollection 2011.
4. Churchland PM. *Neurophilosophy At Work.* 2007. Cambridge Univ Pr ISBN-10: 0521864720. ISBN-13: 978-0521864725
5. Dayan, E., Censor N., Buch, E.R., Sandrini, M., and Cohen, L.G. (2013). Noninvasive brain stimulation: from physiology to network dynamics and back. *Nat. Neurosci.* 16, 838–844. doi: 10.1038/nn.3422
6. de Arcangelis L, Herrmann HJ. 2010. Learning as a phenomenon occurring in a critical state. *PNAS* 107: 3977-3981.
7. Djamanakova A, Tang X, Li X, Faria AV, Ceritoglu C, et al. Tools for Multiple Granularity Analysis of Brain MRI Data for Individualized Image Analysis. *Neuroimage.* 2014 Jun 26. pii: S1053-8119(14)00523-0. doi: 10.1016/j.neuroimage.2014.06.046. [Epub ahead of print]
8. Dodson, C.T.J. and P.E. Parker, *A user's guide to algebraic topology*, Kluwer, Dordrecht, Netherlands, 1997, xii+405 pp. ISBN: 0-7923-4292-5,MR1430097.
9. Ehresmann, C. (1950). Les connexions infinitésimales dans un espace fibrée différentiable. *Colloque de Topologie*, Bruxelles, pp. 29–55.
10. Esposito U, Giugliano M, van Rossum M, Vasilaki E. Measuring Symmetry, Asymmetry and Randomness in Neural Network Connectivity. *PLoS One.* 2014 Jul 9;9(7):e100805. doi: 10.1371/journal.pone.0100805.

11. Ihlen, E.A. (2012). Introduction to multifractal detrended fluctuation analysis in matlab. *Front. Physiol.* 141, 1-18. doi: 10.3389/fphys.2012.00141
12. Freeman WJ, Vitiello G. Dissipation and spontaneous symmetry breaking in brain dynamics. *J. Phys. A: Math. Theor.* 41 (2008) 304042, (pp. 1-17). doi:10.1088/1751-8113/41/30/304042
13. Friston K (2010). The free-energy principle: a unified brain theory? *Nature Reviews Neuroscience.* 11:127-138. doi:10.1038/nrn2787.
14. Friston, K. (2013) Life as we know it. *Journal of The Royal Society Interface* 10
15. Friston, K., M. Breakspear and G. Deco (2012). "Perception and self-organized instability." *Frontiers in Computational Neuroscience* 6: 44.
16. Friston KJ, Kahan J, Razi A, Stephan KE, Sporns O. On nodes and modes in resting state fMRI. *Neuroimage.* 2014 May 24. pii: S1053-8119(14)00421-2. doi: 10.1016/j.neuroimage.2014.05.056.
17. Garfinkel SN, Minati L, Gray MA, Seth AK, Dolan RJ, Critchley HD. Fear from the heart: sensitivity to fear stimuli depends on individual heartbeats. *J Neurosci.* 2014 May 7;34(19):6573-82. doi: 10.1523/JNEUROSCI.3507-13.2014.
18. Gollisch T (2009). Throwing a glance at the neural code: rapid information transmission in the visual system. *HFSP J* 3(1): 36–46. doi: 10.2976/1.3027089
19. Haider B, Duque A, Hasenstaub AR, McCormick DA. Neocortical network activity in vivo is generated through a dynamic balance of excitation and inhibition. *J Neurosci.* 2006 Apr 26;26(17):4535-45.
20. Higgs PW. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.* 13, 508 – Published 19 October 1964. DOI: <http://dx.doi.org/10.1103/PhysRevLett.13.508>
21. Huth AG, de Heer WA, Griffiths TL, Theunissen FE, Gallant JL. Natural speech reveals the semantic maps that tile human cerebral cortex. *Nature.* 2016 Apr 28;532(7600):453-8. doi: 10.1038/nature17637.
22. Jarman N, Trengove C, Steur E, Tyukin I, van Leeuwen C. Spatially constrained adaptive rewiring in cortical networks creates spatially modular small world architectures. *Cognitive Neurodynamics.* April 2014. DOI: 10.1007/s11571-014-9288-y
23. Koch C, Massimini M, Boly M, Tononi G. Neural correlates of consciousness: progress and problems. *Nat Rev Neurosci.* 2016 Apr 20;17(5):307-21. doi: 10.1038/nrn.2016.22.
24. Jiang 2015

25. Lewin K. A dynamic theory of personality. McGraw_Hill Book Company, Inc. New York and London, 1935.
26. Lombardi F, Herrmann HJ, Perrone-Capano C, Plenz D, de Arcangelis L. Balance between excitation and inhibition controls the temporal organization of neuronal avalanches. *Phys Rev Lett.* 2012 Jun 1;108(22):228703. Epub 2012 May 31.
27. Ma C, Pan X, Wang R, Sakagami M. Estimating causal interaction between prefrontal cortex and striatum by transfer entropy. *Cogn Neurodyn.* 2013 Jun;7(3):253-61. doi: 10.1007/s11571-012-9239-4. Epub 2013 Jan 4.
28. Matsui T. Gauge Symmetry and Neural Networks. in "Fluctuating Paths and Fields"ed. by W.Janke et al., World Scientific (2001) pp. 271-280
29. Mattei, T.A. (2014). Unveiling complexity: non-linear and fractal analysis in neuroscience and cognitive psychology. *Front. Comput. Neurosci.* 21; 8:17. doi: 10.3389/fncom.2014.00017
30. McDonough IM, Nashiro K. Network complexity as a measure of information processing across resting-state networks: evidence from the Human Connectome Project. *Front Hum Neurosci.* 2014 Jun 10;8:409. doi: 10.3389/fnhum.2014.00409. eCollection 2014.
31. Milstein, J., Mormann, F., Fried, I., and Koch, C. (2009). Neuronal shot noise and Brownian 1/f² behavior in the local field potential. *PLoS One.* 4, e4338. doi: 10.1371/journal.pone.0004338
32. Mosso A. Fear. Longmans, Green and Co. London New york, and Bombay. 1896
33. Nieuwenhuys R, Voogd J, van Huijzen C (2008) The Human Central Nervous System. Springer, Heidelberg
34. Xue M, Atallah BV, Scanziani M. Equalizing excitation–inhibition ratios across visual cortical neurons. *Nature* (2014) doi:10.1038/nature13321
35. Opris I, Casanova MF. Prefrontal cortical minicolumn: from executive control to disrupted cognitive processing. *Brain.* 2014 Jul;137(Pt 7):1863-75. doi: 10.1093/brain/awt359. Epub 2014 Feb 14.
36. Papo D. Functional significance of complex fluctuations in brain activity: from resting state to cognitive neuroscience. *Front. Syst. Neurosci.*, 11 June 2014 | doi: 10.3389/fnsys.2014.00112
37. Park, H.D., Correia, S., Ducorps, A., Tallon-Baudry, C. (2014). Spontaneous fluctuations in neural responses to heartbeats predict visual detection. *Nat. Neurosci.* 2014 Mar 9. [Epub ahead of print]. doi: 10.1038/nn.3671

38. Peters JF. 2016. Computational Proximity. Excursions in the Topology of Digital Images. Edited by Intelligent Systems Reference Library. Berlin: Springer-Verlag. doi:10.1007/978-3-319-30262-1.
39. Peters JF, Tozzi A. 2016. "Region-Based Borsuk-Ulam Theorem." *arXiv*.1605.02987
40. Priesemann V. Self-organization to sub-criticality. *BMC Neuroscience* 2015, 16(Suppl1):O19 <http://www.biomedcentral.com/1471-2202/16/S1/O19>
41. Pritchard, W.S. (1992). The brain in fractal time: 1/f-like power spectrum scaling of the human electroencephalogram. *Int. J. Neurosci.* 66, 119-129.
42. Sengupta B, Stemmler MB, Friston KJ. Information and efficiency in the nervous system--a synthesis. *PLoS Comput Biol.* 2013;9(7):e1003157. doi: 10.1371/journal.pcbi.1003157. Epub 2013 Jul 25.
43. Sengupta, Biswa, Arturo Tozzi, Gerald K. Cooray, Pamela K. Douglas, and Karl J. Friston. 2016a. "Towards a Neuronal Gauge Theory." *PLOS Biology* 14 (3): e1002400. doi:10.1371/journal.pbio.1002400.
44. Sengupta B, Friston KJ, Penny WD. 2016b Gradient-based MCMC samplers for dynamic causal modelling. *Neuroimage.* 2016 Jan 15;125:1107-18. doi: 10.1016/j.neuroimage.2015.07.043. Epub 2015 Jul 23.
45. Smith MA, Scholey AB (2014) Nutritional influences on human neurocognitive functioning. *Front Hum Neurosci* 8:358. doi: 10.3389/fnhum.2014.00358
46. Snaider J, Franklin S. "Vector LIDA." *Procedia Computer Science* 41 (2014): 188-203.
47. Sporns O (2013). Network attributes for segregation and integration in the human brain. *Curr Opin Neurobiol.* 23(2):162-71. doi: 10.1016/j.conb.2012.11.015.
48. Sreekumar V, Dennis S, Doxas I, Zhuang Y, Belkin M. The geometry and dynamics of lifelogs: discovering the organizational principles of human experience. *PLoS One.* 2014 May 13;9(5):e97166. doi: 10.1371/journal.pone.0097166. eCollection 2014.
49. 't Hooft G. Renormalizable Lagrangians for massive Yang-Mills fields. *Nuclear Physics B.* Volume 35, Issue 1, 1 December 1971, Pages 167–188
50. Taylor P, Hobbs JN, Burrioni J, Siegelmann HT. The global landscape of cognition: hierarchical aggregation as an organizational principle of human cortical networks and functions. *Sci Rep.* 2015 Dec 16;5:18112. doi: 10.1038/srep18112.
51. Tognoli E, Scott Kelso JA. Enlarging the scope: grasping brain complexity. *Front. Syst. Neurosci.*, 25 June 2014. doi: 10.3389/fnsys.2014.00122

52. Tononi G. Consciousness as integrated information: a provisional manifesto. *Biol Bull.* 2008 Dec;215(3):216-42.
53. Tozzi A. 2015. Information Processing in the CNS: A Supramolecular Chemistry? *Cognitive Neurodynamics* 9 (5): 463–77.
54. Tozzi A, Peters JF. 2016a. “A Topological Approach Unveils System Invariances and Broken Symmetries in the Brain.” *Journal of Neuroscience Research* 94 (5): 351–65. doi:10.1002/jnr.23720.
55. Tozzi, A, Peters JF. 2016b. Towards a Fourth Spatial Dimension of Brain Activity. *Cognitive Neurodynamics* 10 (3): 189–99. doi:10.1007/s11571-016-9379-z.
56. Tozzi A, Fla Tor, Peters JF. 2016. “Building a Minimum Frustration Framework for Brain Functions in Long Timescales.” *J Neurosci Res.* doi:10.1002/jnr.23748.
57. Van Essen, D.C. (2005). A Population-Average, Landmark- and Surface-based (PALS) atlas of human cerebral cortex. *Neuroimage.* 28, 635–666.
58. Whitehead AN. An enquiry concerning the principles of natural knowledge. Cambridge at the University Press, 1919.
59. Wu X, Foster DJ. Hippocampal replay captures the unique topological structure of a novel environment. *J Neurosci.* 2014 May 7;34(19):6459-69. doi: 10.1523/JNEUROSCI.3414-13.2014.
60. Yoon K, Buice MA, Barry C, Hayman R, Burgess N, Fiete IR. Specific evidence of low-dimensional continuous attractor dynamics in grid cells. *Nat Neurosci.* 2013 Aug;16(8):1077-84. doi: 10.1038/nn.3450. Epub 2013 Jul 14.
61. Zeidler, E. (2011). *Quantum Field Theory III: Gauge Theory.* Springer. doi: 10.1007/978-3-642-22421