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Letter to Editor

How to turn an oscillation in a pink one

HIGHLIGHTS

- Scale-free dynamics are an intrinsic feature of a large class of natural models.
- A fractal-like wave can be produced by summing a random oscillation to a chosen one.
- Fractal systems can be produced by choosing the appropriate oscillation to bring in.

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Scale-free dynamics – also called $1/f^n$ behavior, pink noise, power law, fractal-like distribution (Newman, 2005) – are an intrinsic feature of a large class of natural models, from earthquakes to brain activity (He et al., 2010). Assessing a geometrical/mathematical model of synthetic power law oscillations, we noticed that a wave containing a fractal-like structure can be produced by summing a random oscillation to a carefully chosen one. Here follows the procedure we carried out. First, we generated random numbers with power law distribution, obtaining a series of oscillations **1** in a log amplitude versus log frequency scatter plot (the solid line in Fig. 1A). The series exhibited a pink noise behavior: $A(f)=1/f^n$, where A was the amplitude of the oscillation, f was the frequency and n was the power exponent (Milstein et al., 2009). Second, we produced a non scale-free oscillation **2** (the dotted line in Fig. 1A) equipped with a random point $\mathbf{O_2}$. Third, we chose the point $\mathbf{O_1}$ characterized by the same frequency of $\mathbf{O_2}$, but a different amplitude. Fourth, we extrapolated the sine waves corresponding to the points $\mathbf{O_1}$ and $\mathbf{O_2}$ (Fig. 1B). Fifth, we looked for the required oscillation $\mathbf{O_3}$ which we had to superimpose upon $\mathbf{O_2}$, in order that the combination might be equal to the oscillation $\mathbf{O_1}$. We handled the sinusoidal oscillations through the classical formula (Booker, 1993) reported in Fig. 1B, where t denotes time.

In summary, we demonstrated that a pink noise can be obtained by adding a random sine wave to a proper one. This observation gives rise to countless applications: a “hidden” oscillation may cause a scale-free behavior in a random noise; a fractal system can be produced by simply choosing the appropriate oscillation to bring in; if power laws are involved in random walks, phase transitions and self-organized criticality (Bak et al., 1987), then the superimposition of a carefully chosen oscillation may lead to systems of increased complexity; “nested” waves from the central nervous system's spontaneous networks (Fox and Raichle, 2007) may be the source of the scale-free dynamics seen in EEG and fMRI; in the event of brain 1/f scaling disruption caused by illnesses such as Alzheimer's disease, an external wave – for instance, via transcranial stimulation (Reato et al., 2013) – could restore the broken symmetry.

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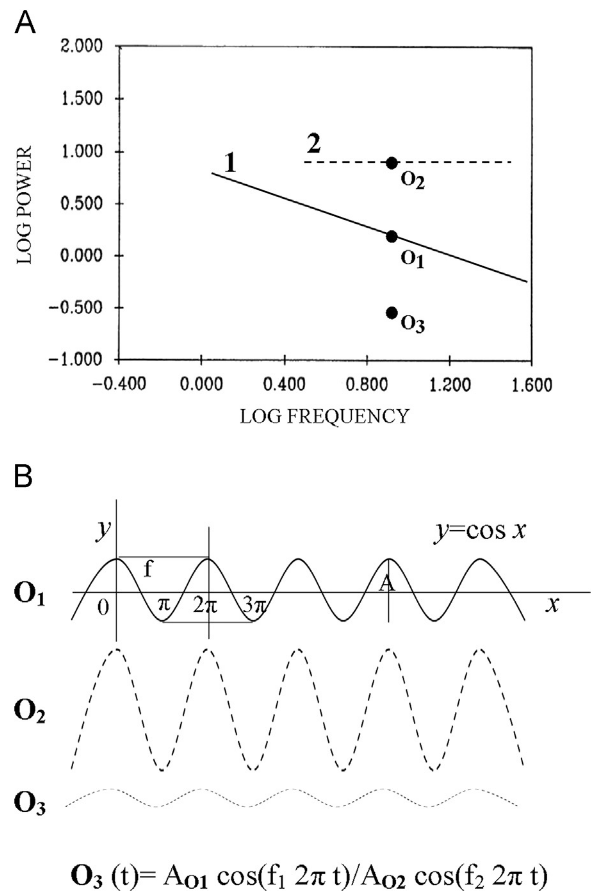

Fig. 1A. Power law distributions in a log power (amplitude) versus log frequency scatter plot. Fig 1B. Sine waves corresponding to the points O_1 and O_2.

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